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Chemistry. — *Membrane and Osmosis.* II. By F. A. H. SCHREINEMAKERS.

(Communicated at the meeting of October 26, 1929).

Ternary liquids and a membrane M(W).

If we bring a membrane $M(W)$ into a liquid L, we can represent its composition by :

$$1 \text{ gr. } Q + w \text{ mol } W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the preceding communication [1]) we have taken a binary liquid for L ; now we shall take a ternary liquid for L. When this liquid contains the substances $X + Y + W$ we may represent its composition in the usual way by a point of the triangle WXY ; in the figs 1 and 2 this triangle has been drawn perspectively and in an other position than usual.

We now represent the W-amount, a liquid z gives to a membrane, by the point z' viz. by the length of the line zz' perpendicular to the plane of the triangle WXY.

When this liquid z gets all possible compositions, so that point z travels over the entire triangle, then point z' travels over a plane, which we shall call the W-plane of the membrane. This W-plane is bounded :

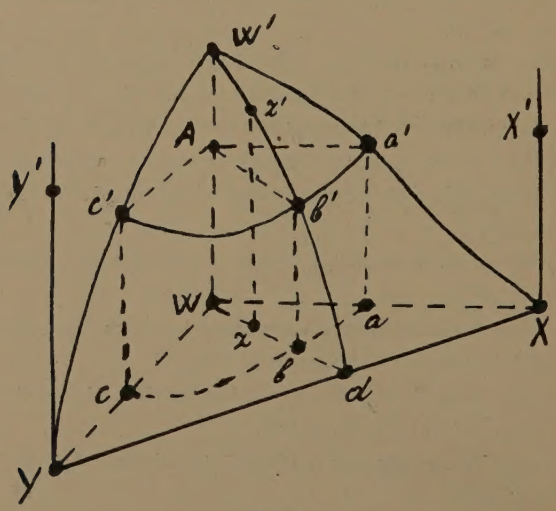

Fig. 1.

[1]) These Proceedings 32, 837, (1929).

in the plane $W'WXX'$ by a curve $W'a'X$; this is the W-curve of the membrane for the liquids, consisting of $W + X$.

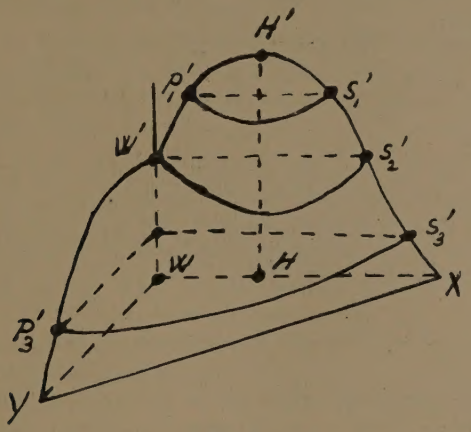

Fig. 2.

in the plane $W'WYY'$ by a curve $W'c'Y$; this is the W-curve of the membrane for the liquids, consisting of $W + Y$.

in the horizontal plane by the side XY of the triangle WXY.

The vertical plane $W'Wd$ (fig. 1) intersects the W-plane along a curve $W'z'b'd$; the points of this curve represent the W-amount, the membrane gets in the liquids of the line $Wzbd$.

In Comm. M. O. I we have deduced what has been stated in B with the aid of A_1 and A_2, namely:

the W-curve of a membrane $M(W)$ is monotonous.

In a corresponding way we can deduce now that the intersection $W'z'b'd$ is monotonous; so we find:

A_1. every flat plane, going through the axis $W'W$, intersects the W-plane of a membrane $M(W)$ according to a monotonous curve.

We shall express this more simply by:

A_2. the W-plane of a membrane $M(W)$ is monotonous.

Ternary liquids and a membrane $M(X)$ or $M(Y)$.

All that has been said above of a membrane $M(W)$ obtains of course also for any other membrane, absorbing one single substance only. If we have a membrane $M(X)$, we may represent its composition by:

$$1 \text{ gr. } Q + x \text{ mol } X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

if we have a membrane $M(Y)$ we may represent its composition by:

$$1 \text{ gr. } Q + y \text{ mol } Y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We now imagine the X-amount of a membrane $M(X)$ represented in fig. 1 in a way corresponding to that of the W-amount of the membrane $M(W)$; we then get a plane, we shall call the X-plane of the membrane $M(X)$.

This X-plane has of course an other position than the W-plane; in fig. 1 namely it goes through the line WY and through a point X', which represents the X-amount, the membrane gets in the pure liquid X.

If we imagine the Y-amount of a membrane $M(Y)$ represented in a similar way, we get the Y-plane of the membrane $M(Y)$; in fig. 1 this plane goes through the line WX and a point Y'.

In a similar way as has been indicated above we now find:

B. the X-plane of a membrane $M(X)$ and the Y-plane of a membrane $M(Y)$ are monotonous.

Ternary liquids and a membrane $M(n)$.

If we bring a membrane $M(n)$ into a ternary liquid, consisting of the substances $W+X+Y$, we can represent the composition of this membrane by:

$$1 \text{ gr. } Q + w \text{ mol } W + x \text{ mol } X + y \text{ mol } Y \quad . \quad . \quad . \quad (4)$$

We now imagine the W-, the X- and the Y-amount, a liquid z gives to this membrane, represented in fig. 1 by the points z', z'' and z''' (the points z'' and z''', which must of course be situated somewhere on line $z\,z'$, have not been indicated in fig. 1).

When this liquid z travels over the triangle WXY, each point describes a plane viz. the W-, the X- and the Y-plane of this membrane $M(n)$.

Consequently the membrane $M(n)$ has a W-, an X- and an Y-plane, while a membrane $M(W)$ has only a W-plane in these liquids, a membrane $M(X)$ only an X-plane and a membrane $M(Y)$ only an Y-plane.

Above we have seen that the W-plane of a membrane $M(W)$, the X-plane of a membrane $M(X)$ and the Y-plane of a membrane $M(Y)$ are monotonous. As we shall see later on, however, this does not always obtain any more for the W-, the X- and the Y-plane of a membrane $M(n)$.

We shall now begin by considering the W-plane; this has two boundary curves; the one is situated in plane $WW'X$ and is the W-curve of the membrane $M(n)$ for the liquids, consisting of $W+X$; the other is situated in plane $W'WY$ and is the W-curve of the membrane $M(n)$ for the liquids, consisting of $W+Y$. We may now distinguish the following cases:

C_1. both boundary curves are monotonous (fig. 1).

C_2. one of the boundary curves is monotonous, the other has a maximum (fig. 2).

C_3. both curves have a maximum.

C_4. in one or in both boundary curves occurs not only a maximum, but also a minimum.

If we have a membrane, for which C_1 obtains, we may imagine the W-plane represented by fig. 1; here not only the two boundary curves are monotonous, but also all sections with a plane, going through the axis WW'.

In pure water the membrane now gets a greater W-amount than in all other binary and ternary liquids.

If we have a membrane for which C_2 obtains, the W-plane may be shaped as has been drawn in fig. 2; the boundary curve $W'Y$ is monotonous, the boundary curve $W'X$ has a maximum in H'.

We now imagine in fig. 2 also an intersection-plane $W'W\,d$ like the one in fig. 1. If we turn this from the position $W'W\,X$ into the position $W'W\,Y$, we shall first get a series of intersections with a maximum and afterwards a series of monotonous intersections; in a certain position of the plane $W'W\,d$ this maximum is situated in the point W'.

In turning the plane $W'W\,d$ we, therefore, get a series of maxima; one of these maxima is highest; this to be sure is the highest point of the W-plane; we may now distinguish two cases:

a. In fig. 2 we have assumed that the maximum H' of the boundary curve $W'X$ is also the highest point of the W-plane. We then have:

in the binary liquid H the membrane gets a greater W-amount than in all other binary and ternary liquids and than in water.

b. It may also be imagined that the highest point of the W-plane is not situated in the boundary plane. If we imagine the horizontal projection of this point represented in fig. 2 by Q, then follows:

in the ternary liquid Q the membrane gets a greater W-amount than in all binary and ternary liquids and than in water.

So from the preceding considerations appear already among other things:

a membrane $M(n)$ gets its greatest W-amount in pure water or in one of the binary or in one of the ternary liquids;

a membrane $M(W)$, however, always gets its greatest W-amount in the water.

The reader will now have no difficulty in deducing what shapes the W-plane of a membrane can have, for which C_3 or C_4 obtains.

All that has been said above of the W-plane, also obtains for the X- and the Y-plane of a membrane $M(n)$. As, therefore, each of the three planes can already have many shapes of its own, their combination will give rise to a great many cases; we shall discuss some of these cases later on.

The iso M-curves.

As we shall see immediately an infinite number of liquids exist, which will give an equal W- or an equal X- or an equal Y-amount to a membrane;

these liquids are situated on a curve which we shall call an "iso M-curve".

When these liquids give an equal W-amount to a membrane $M(W)$, we shall call this curve an "iso $W.M(W)$-curve"; if they give an equal W-amount to a membrane $M(n)$, we shall call this curve an "iso $W.M(n)$-curve".

When these liquids give an equal X-amount (or Y-amount) to a membrane $M(n)$, we shall represent these curves in a similar way. Consequently we have in all 6 kinds of iso M-curves, namely:

iso $W.M(W)$-, iso $X.M(X)$-, iso $Y.M(Y)$-curves
iso $W.M(n)$-, iso $X.M(n)$-, iso $Y.M(n)$-curves.

We shall see later on that the first three curves play quite an other part during the osmosis than the others.

The iso $W.M(W)$-, the iso $X.M(X)$- and the iso $Y.M(Y)$-curves.

As the W-plane of a membrane $M(W)$ is monotonous, we may imagine this to be represented by fig. 1. If here we suppose: $aa' = bb' = cc'$ so that the plane $Aa'b'c'$ will be horizontal, the liquids a, b and c and also all other liquids of curve abc will give the same W-amount to the membrane $M(W)$. From this appears:

curve abc is an iso $W.M(W)$-curve.

This curve divides the triangle into two fields; all liquids within field $Wabc$ (fig. 1) give a greater, all liquids within field $abcYdX$ a smaller W-amount to the membrane than the liquids of curve abc.

Now we see also that an infinite number of these curves exist; if we imagine the plane $Aa'b'c'$ higher (lower), then curve abc will be situated closer to (farther from) point W. From this appears:

D. the horizontal projection of every horizontal intersection of the W-plane of a membrane $M(W)$ is an iso $W.M(W)$-curve.

If we now imagine some iso $W.M(W)$-curves drawn in the triangle WXY of fig. 1, we see among other things:

1. Two curves can never intersect or touch one another.
2. Every straight line, going through point W, intersects a curve in one point only.
3. The W-amount which the liquids of an iso $W.M(W)$-curve give to a membrane, will become larger, the nearer this curve is situated to point W.
4. These curves are straight lines in the vicinity of point W; at a greater distance they are curved and may take various shapes.

These properties quite agree with those of the isotonic W-curves, discussed before [1]; later on we shall see to be sure that iso $W.M(W)$-curves are the same as isotonic W-curves.

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[1] Versl. Kon. Akad. v. Wet. Amsterdam, **37**, 637, (1928); these Proceedings, **31**, 814, (1928).

If in fig. 1 we now imagine also the X-plane of a membrane $M(X)$ or the Y-plane of a membrane $M(Y)$, we shall find in a way corresponding to the one described above, the iso $X.M(X)$- or the iso $Y.M(Y)$-curves.

Of course similar properties obtain for these curves as for the iso $W.M(W)$-curves.

The iso $W.M(n)$-, the iso $X.M(n)$- and the iso $Y.M(n)$-curves.

If we intersect the W-plane of a membrane $M(n)$ by a horizontal plane, then the horizontal projection of this intersection represents all liquids, giving an equal W-amount to the membrane $M(n)$. Instead of D we now have:

E. the horizontal projection of every horizontal intersection of the W-plane of a membrane $M(n)$ is an iso $W.M(n)$-curve.

If we have a membrane $M(n)$ for which the W-plane has a shape as has been drawn in fig. 1, then the same obtains for the position of the iso $W.M(n)$-curve with respect to point W as for the iso $W.M(W)$-curves, discussed above. All that has been said above in 1—4 of the iso $W.M(W)$-curves, now also obtains for these iso $W.M(n)$-curves; however, but these are not isotonic W-curves, as we shall see later on.

We now imagine a membrane $M(n)$ for which the W-plane has a shape like that in fig. 2. If we project the horizontal intersections $p'_1 s'_1$, $W' s'_2$ and $p'_3 s'_3$ on the triangle WXY, we can imagine these projections represented by the curves $p_1 s_1$, $W s_2$ and $p_3 s_3$ of fig. 2 in Comm. [1]) Gen. VIII.

This figure shows among other things:

all liquids of curve $Wqrs_2$ give the same W-amount to the membrane as the pure water;

all liquids within the field $Wqrs_2 HW$ give a larger W-amount to the membrane than the pure water; of all these liquids the binary liquid H gives the largest W-amount to the membrane;

all liquids outside field $Wqrs_2 HW$ give a smaller W-amount to the membrane than pure water.

Now it appears from fig. 2 of this paper and fig. 2 of Comm. Gen. VIII, that with this iso $W.M(n)$-curve point H plays a part corresponding to that of point W with the preceding curves.

All that has been said above in 1—4 of the iso $W.M(W)$-curves, now also obtains for these iso $W.M(n)$-curves, provided we replace point W in 1—4 by point H; in the vicinity of point H, however, these curves will not be straight, but parabolic.

We have seen above that there are also W-planes, of which the highest point Q' is not situated in one of the boundary planes; the projection Q of this point Q' will then be situated within the triangle WXY. Then there

[1]) These Proceedings, **32**, 29, (1929).

are iso $W.M(n)$-curves, surrounding point Q and situated entirely within the triangle. In 1—4 point W should now be replaced by point Q; in the vicinity of this point the curves will then be elliptical.

The above shows among other things:

F_1. for all iso $W.M(W)$-curves the properties 1—4 obtain;

F_2. for the iso $W.M(n)$-curves the properties 1—4 obtain also, but point W may or may not have been replaced by a binary or ternary point (H or Q). It depends on the nature of the membrane and on the substances X and Y, wich of these three cases will occur.

Corresponding properties as for the iso $W.M(n)$-curves obtain of course for the iso $X.M(n)$- and iso $Y.M(n)$-curves.

The absorption by a membrane.

If a membrane that has not yet absorbed strange substances, is brought into a liquid, the composition of the membrane will first change rapidly and afterwards slowly; at last follows a series of very slow changes, which may go on for months and years.

Perhaps it is allowed to imagine that during the first period the liquid enters into the intermicellary spaces, pores and little channels and fills them; at the same time the micelles also begin to absorb some of the liquid that has entered into the membrane. Under the influence of the capillary powers this said liquid may have an other composition than the liquid outside the membrane.

After all intermicellary spaces have been filled, it will of course take some time more before all micelles have also been saturated; as this absorption will generally take place much more slowly than the filling of the intermicellary spaces, the second period will naturally last much longer than the first.

In the third period a part is played by the colloidal properties of the membrane, which were to be sure also active in the first and second periods. All sorts of factors, viz. the praehistory of the membrane, the influence of the absorbed substances, etc. may change the built, the structure, the density, etc. of the micelles and the dimensions of the intermicellary spaces; the membrane certainly changes its composition. As a rule these changes take place only very slowly and perhaps they do not stop until the membrane has lost its colloidal properties.

Whereas with not-colloidal substances we can speak of a system in equilibrium, we can do so no longer when the system contains a membrane. Then we might say indeed that a series of conditions exists, determined at any moment by the condition in which the membrane is found at that moment, which condition is determined again by the praehistory and the influence of the absorbed substances.

All this shows clearly that the above discussed W-curve, W-plane etc. of a membrane, do not represent states of equilibrium; theoretically they have different shapes every moment, although they may practically remain unchanged during a shorter or a longer time. Besides it follows from the way in which these curves and planes have to be determined experimentally, that they cannot give anything but an approximate image of their shapes, which may vary in many respects from their true shapes. We shall refer to this later on, when discussing some examples.

(To be continued.)

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Geology. — *Les Roches de Tahiti.* Par W. F. GISOLF.

(Communicated at the meeting of October 26, 1929.)

Il y a quelque temps, j'ai introduit [1]) dans la science pétrologique les projections du tetraèdre de M. NIGGLI sur ses plans bisecteurs ; employant chaque fois deux de ces plans mutuellement perpendiculaires ; j'ai déformé les triangles isocèles résultants en triangles rectangles, pour l'usage de papier millimétré. J'y ai ajouté, en suivant les suggestions de M. BECKE [2]), un diagramme pour le silice.

L'emploi du tetraèdre de NIGGLI, qu'on accepte sa classification ou non, a des avantages que nul autre méthode ne possède. L'inconvénient du triangle d'Osann, la projection de l'acide silicique avec les oxydes basiques, a été évité par M. NIGGLI, qui ne projete dans son tetraèdre que les oxydes basiques. J'ai publié plusieurs articles [3]) sur l'emploi de ces projections, mettant en lumière les grands avantages ; dont le plus grand est qu'en n'utilisant aucune classification particulière, on met l'analyse chimique en relation avec les minéraux modaux et normatifs. Avec cette méthode j'ai réussi à démontrer que les magmes de l'Etna en Sicile et du Katmai en Alaska, ont été des roches dioritiques solides, qui ont été refondues (voir mon article sur le Katmai).

Dans le présent article je me propose d'examiner les roches de Tahiti, dont M. A. LACROIX a fait une étude approfondie [4]). Elles réprésentent un des cas des plus belles de différenciation magmatique du monde, étant eloignées loin des massifs continentaux. J'emploierai les projections du tetraèdre sur les deux plans bisecteurs, passant par les arêtes c-al et fm-alk (fig. 1, 2, 3), parce que ces projections permettent non seulement d'étudier le mieux la différenciation magmatique, mais aussi de tirer des conclusions sur la composition du magme originel (ou plutôt, comme nous le verrons, des magmes originels) et sur le volcanisme plus profond de cette région.

On trouvera les noms des roches et leurs paramètres niggliens dans la Table I ; les graphiques correspondantes dans les figures 1, 2 et 3.

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[1]) W. F. GISOLF, Zur graphischen Darstellung von Gesteinsanalysen T. M. P. M. **39**, p. 76, 1928.

[2]) F. BECKE, Graphische Darstellungen von Gesteinsanalysen. T.M.P.M. **37**, p. 27, 1925.

[3]) W. F. GISOLF, Over het verschil in samenstelling etc. Versl. Kon. Akad. v. Wet, Amsterdam, 1927, p. 1025.

————, Over de verhouding tusschen de talische en de femische mineralen etc. Ibid. 1928, p. 56.

————, Een nieuwe functie φ in stollingsgesteenten. Ibid. 1928, p. 271.

————, Die Laven vom Etna. These Proceedings, Vol. **31**, p. 660, 1928.

————, The Rocks of the Katmai Region. Ibid. Vol. **32**, p. 930, 1929.

[4]) A. LACROIX, La constitution lithologique des îles de la Polynésie australes. Mémoires de l' Académie des Sciences. Paris 1927. Tome LIX.

TABLE DES PARAMÈTRES NIGGLIENS DES ROCHES DE TAHITI.

N°.[1])	Nom des Roches	*si*	$2fe_2O_3$	*feo*	*mgo*	*c*	*al*	*na*	*ka*
	Roches intrusives.								
1	Syénite néphélinique	173	4.8	5.7	3.4	7.1	41.0	26.1	11.9
2	Syénite néphélinique	160	7.8	8.4	6.3	13.5	33.4	21.3	9.3
3	Monzonite néphélinique	158	5.1	6.2	3.9	12.9	41.0	22.3	8.6
4	Monzonite néphélinique	148	7.9	10.1	8.8	12.4	35.2	18.2	7.4
5	Monzonite néphélinique	105	6.9	12.7	14.8	23.2	25.1	12.7	4.6
6	Essexite	126	6.7	10.5	14.3	17.9	31.2	13.5	5.9
7	Essexite	120	8.4	11.3	11.7	19.2	31.9	12.7	4.8
8	Essexite	125	7.0	12.3	12.5	22.1	28.5	12.6	5.0
9	Syénite néphélinique	164	6.0	9.6	8.5	12.8	34.5	15.4	13.2
10	Syénite pyritisée	276	7.6	0.0	4.9	3.5	42.6	23.0	18.4
11	Microgabbro	93	7.5	14.8	30.5	24.1	16.8	5.0	1.3
12	Luscladite	114	10.7	16.3	13.6	25.2	21.9	9.2	3.1
13	Bérondrite	87	9.5	13.9	24.4	26.0	17.9	5.9	2.4
14	Bérondrite	90	8.4	13.4	24.7	25.7	17.3	7.5	3.0
15	Bérondrite	89	8.3	12.1	23.7	29.0	17.4	6.9	2.6
16	Bérondrite	84	6.7	12.3	28.4	23.4	19.9	6.0	3.3
17	Bérondrite	70	10.8	13.6	21.8	30.0	17.8	4.7	1.3
18	Mafraïte	88	12.3	12.3	29.9	23.5	14.3	5.8	1.9
19	Microbérondrite	78	7.7	13.8	29.7	35.1	12.3	0.9	0.5
20	Yamoskite péridotique	70	10.4	12.8	37.7	29.5	7.7	1.4	0.5
21	Diopsidite à olivine	79	7.1	9.3	55.8	23.8	3.3	0.6	0.1
22	Mafraïte à olivine et biotite	83	10.4	12.9	25.4	27.8	16.9	5.4	1.2
	Roches filoniennes.								
23	Essexite doléritique	126	6.8	13.6	16.7	19.3	27.9	11.0	4.7
24	Camptonite	159	7.1	12.7	10.3	10.5	32.8	19.8	6.7
25	Camptonite	143	6.8	12.0	11.8	13.6	32.7	17.4	5.7
26	Camptonite	122	7.8	9.1	12.6	18.6	30.8	15.6	5.5
27	Camptonite	86	10.5	11.9	28.2	24.4	15.6	6.1	3.3
28	Camptonite	104	10.2	15.5	16.0	23.1	21.9	10.5	2.8
29	Fourchite	105	7.2	12.3	20.0	22.7	23.2	12.1	2.5
30	Monchiquïte	100	9.2	13.4	18.9	28.5	20.2	8.0	1.8

[1]) Ces numéros correspondent avec les numéros des analyses données par M. LACROIX. Op. cit. pg. 8—22.

TABLE DES PARAMÈTRES NIGGLIENS DES ROCHES DE TAHITE.
(*Continuation*)

N°.	Nom des Roches	*si*	$2fe_2O_3$	*feo*	*mgo*	*c*	*al*	*na*	*ka*
	Roches volcaniques.								
31	Trachyte phonolitique	232	6.4	5.0	5.0	5.2	39.8	25.0	13.6
32	Trachyte phonolitique	226	3.6	5.8	6.7	7.0	39.9	26.2	10.8
33	Trachyte phonolitique	223	6.2	4.2	4.2	6.2	42.0	23.3	13.9
34	Phonolite	210	7.3	2.3	3.2	5.8	42.5	25.9	13.0
35	Phonolite à haüyne	151	7.8	9.7	11.6	13.6	28.7	21.6	7.0
36	Tinguaïte	193	2.9	4.3	2.7	3.5	43.1	31.8	11.7
37	Tahitite	136	4.3	11.8	8.7	18.3	31.3	18.8	6.8
38	Tahitite	139	5.1	11.3	6.7	18.4	31.9	21.3	6.3
39	Tahitite	130	9.8	8.4	9.3	18.8	31.4	15.9	6.4
40	Tahitite	128	7.2	10.1	7.6	16.2	31.2	21.1	6.6
41	Tahitite	133	6.9	14.1	10.7	17.1	27.4	18.6	5.2
42	Tahitite	146	9.8	7.0	10.8	12.0	32.3	20.1	8.0
43	Tahitite néphélinique	170	7.4	6.3	7.4	11.7	38.3	18.0	10.9
44	Andésite	125	12.3	12.0	13.8	23.3	27.5	8.3	2.8
45	Basalte andésitique	100	13.0	9.1	29.4	24.4	16.0	6.5	1.6
46	Basanitoïde	116	8.3	12.1	15.7	18.4	27.1	13.3	5.1
47	Basanitoïde aphyrique	108	6.6	17.5	17.3	22.9	23.7	10.3	1.7
48	Basanitoïde porphyrique	97	17.5	6.4	18.6	24.4	22.3	7.6	3.2
49	Basanitoïde porphyrique	90	5.9	15.9	23.4	28.8	17.4	6.2	2.4
50	Basanitoïde porphyrique	85	5.7	14.7	35.3	25.5	13.9	3.5	1.4
51	Basanitoïde	86	6.0	14.7	31.5	27.0	14.6	5.1	1.1
52	Basanitoïde doléritique	87	2.9	17.2	40.6	20.7	10.9	6.8	0.9
53	Ankaramite basanitique	77	9.6	9.9	44.0	23.9	9.0	2.5	1.1
54	Ankaramite basanitique	73	5.9	9.8	46.8	26.7	8.2	2.4	0.2
55	Ankaramite océanite	78	5.6	12.9	50.6	19.3	9.2	1.8	0.6
56	Ankaramite océanite	71	4.4	11.7	55.4	18.1	7.5	2.4	0.5
57	Ankaratrite porphyrique	72	3.7	15.1	48.9	18.2	8.6	4.6	0.9
58	Ankaratrite étinditique	81	3.0	17.1	35.3	27.6	10.3	5.2	1.5

J'examinerai d'abord les roches volcaniques, (fig. 1 et 2) parce que, comme on le voit dans ces figures d'une manière extrêmement claire, la différenciation magmatique est plus prononcée dans les roches volcaniques que dans les roches intrusives et filoniennes ; chose facile à comprendre, parce que les laves, une fois écoulées, ne peuvent plus entrer en réaction

avec le contenu de la chambre magmatique dont elles proviennent ; elles sont comme des échantillons documentaires de l'état du magma au moment de son épanchement. Aussi y-a-t-il de bonnes raisons pour nier l'absolu du principe, longtemps considéré comme un dogme, qu'à chaque lave superficielle

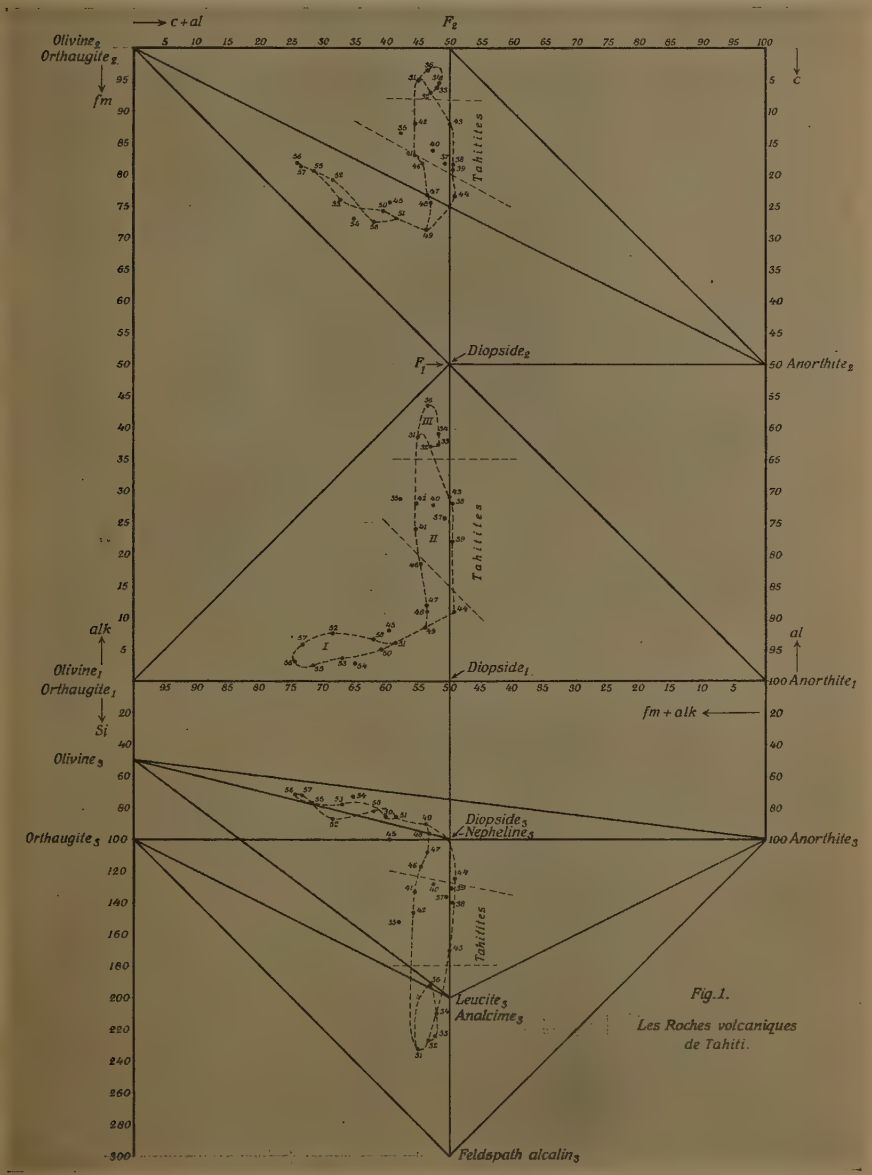
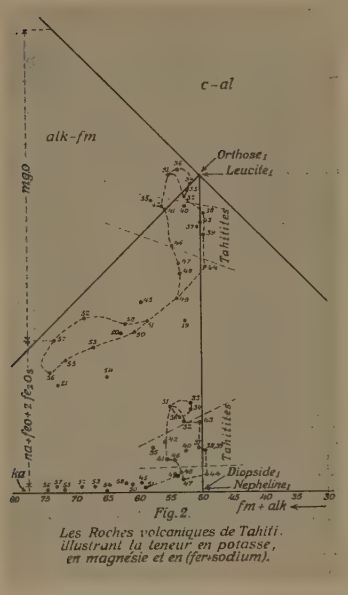


Fig. 1.

Les Roches volcaniques de Tahiti.

doit correspondre une roche intrusive de chimisme analogue et réciproquement. Surtout il n'est pas nécessaire, qu'il y ait eu une seule chambre magmatique ; plusieurs chambres magmatiques, remplies d'abord par un magme de même composition, mais s'ouvrant à des époques différentes quant à leur différenciation, donneront par leurs laves épanchées une image bien supérieure de cette différenciation, qu'une seule chambre ne saurait la donner.



Il n'est pas nécessaire d'employer les coefficients niggliens *k* et *mg* ; dans mon article publié dans les *Tschermaks Mitteilungen* (op. cit.) j'ai mis en lumière qu'un seul système de projections suffit pour tous les composés chimiques ; c'est seulement pour éviter l'accumulation des points, que j'ai dessiné la figure 2 (en même projection que la figure 1) laquelle représente la teneur en magnésie, en potasse et en fer augmenté du sodium.

A l'aide de ces figures on peut distinguer trois phases I, II et III, savoir :

I. Phase basique, formée par les roches 51—58 parmi lesquelles le N⁰. 54 a une position à part à cause de sa teneur en magnésie.

II. Phase acide, formée par les roches 37—49.

III. Phase ultérieure, formée par les roches 31, 32, 33, 34, 36. Il n'est pas difficile de reconnaître que ces roches-ci sont les produits ultérieurs du magme (le „residual liquor" des anglais).

Les roches 45, 35 ont une position à part.

Quand on veut rechercher la composition originelle du magma, on doit tenir compte de la probabilité ou plutôt de la possibilité, que ce magme soit écoulé avant que la différenciation n'ait commencée. Etant entièrement, ou à peu près liquide, ce magme se solidifiera, à sa venue au jour, sous la forme d'une roche vitreuse aphyrique, contenant peut-être quelques cristaux du minéral le plus insoluble, l'olivine. Cette roche est représentée a Tahiti par le N⁰. 45 ; et l'on voit dans les graphiques qu'elle a une position bien marqueé, entre les phases basiques et acides mais, plus près de la phase basique.

Je prétends que la roche N⁰. 45 représente la composition originelle du magma (ou à peu près des magmes) des roches de Tahiti. Je donnerai plus bas quelques indications pour le démontrer ; la preuve absolue pouvant être donnée quand les relations mutuelles des roches dans l'île seront exactement connues, ce qui n'est pas encore le cas.

On trouvera les projections des roches intrusives et filoniennes dans la fig. 3 ; j'y ai dessiné aussi les contours des roches volcaniques de la figure 1 et en outre les roches volcaniques 35 et 45. On voit qu'il y a

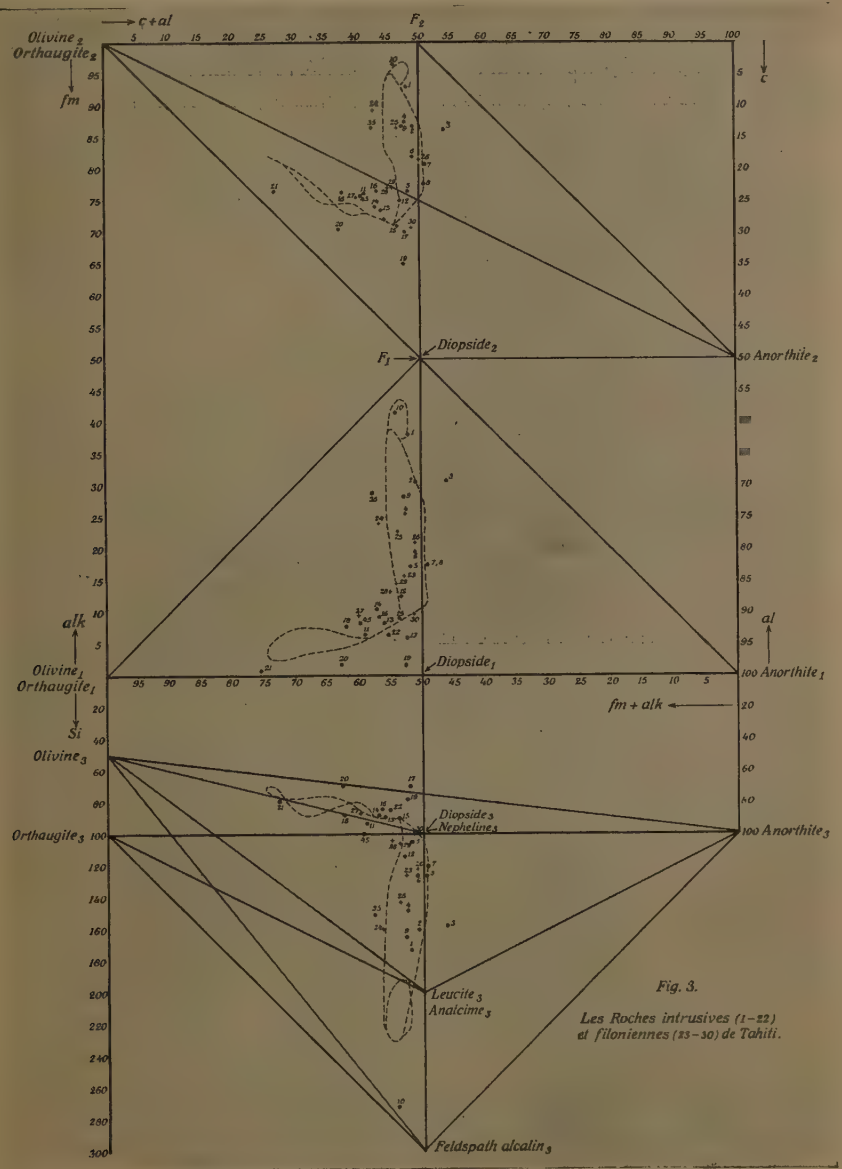


Fig. 3.

Les Roches intrusives (1–22) et filoniennes (23–30) de Tahiti.

beaucoup de roches intrusives et filoniennes qui n'ont pas d'équivalents superficiels, savoir :

1°. Les roches 21, 20, 19, 17 ; ce qui est dû, probablement à ce que les laves n'étaient pas assez liquides pour atteindre la surface, étant trop chargées de cristaux ; ou, à ce qu'étant solides, elles ne pouvaient pas se reliquéfier sous l'influence d'une diminution de pression, la température des magmas étant trop basse pour la liquéfaction des minéraux tel que l'olivine, le diopside, des diopsides magnésiens [1]) et leurs mélanges ou leurs substituts.

2°. Les roches 18, 27, 14, 16, 13 ; ces roches ne diffèrent pas beaucoup du N°. 45 ; il est possible qu'elles se soient solidifiées avant que la différenciation magmatique ait fait des progrès importants ; cette solidification étant probablement due aux dimensions de leur chambre magmatique.

Parmi toutes les roches de Tahiti, indubitablement alcalines, il en est une qui attire l'attention, parce qu'elle est tout à fait étrangère. C'est le numéro 44, une andésite commune [2]). Si je veux prouver que le N°. 45 correspond au magme originel il me faut expliquer d'abord comment cette roche a pu naître dans une pareille famille. Je prétends que la roche 44 était la liquide mère de la roche N°. 18 (mafraïte). En ajoutant sept fois les paramètres niggliens du N°. 44 à une fois les paramètres du N°. 18 et en divisant le résultat par 8, on obtient (les deux oxydes de fer sont pris ensemble) :

a.	*si*	$2fe_2o_3+feo$	*mgo*	*c*	*al*	*na*	*ka*
$\frac{7\times \text{N°. } 44+1\times \text{N°. } 18}{8}$:	93	24.6	27.9	23.5	15.9	6.1	2.0
N°. 45 :	100	22.1	29.4	24.4	16.0	6.5	1.6

On voit que le magma ayant la composition chimique du N°. 45 a bien pu donner l'andésite 44 et la mafraïte N°. 18 ; il pourrait même donner une dacite, 100 étant plus grand que 93. On voit aussi que les valeurs concernant le fer, la magnésie et les alcalies diffèrent un peu ; dans le magma commun des roches 44 et 18, la teneur en fer et en potasse est plus forte, tandis que la teneur en magnésie est plus faible que dans le N°. 45. On remarquera aussi que la teneur en calcium et en sodium diffère; elle est plus faible pour les deux éléments dans le magma commun des roches 44 et 18, que dans le magma du N°. 45. Tout cela a une signification bien importante, comme nous le verrons tout à l'heure.

Cette observation sur l'origine de l'andésite N°. 44 jette une lumière inattendue sur la question de l'origine des roches alcalines et calco-

[1]) Voir les théories de BOWEN sur la possibilité de laves, correspondantes aux péridotites et aux pyroxénolites dans: BOWEN. The evolution of igneous rocks. Princeton 1928. La roche N°. 54 n'est pas en accord avec la théorie de BOWEN.

[2]) J'appellerai des roches pareilles des apatalites; ce qui n'est pas juste envers la roche, mais juste envers la Science.

alcalines en général. Aussi je me propose de publier, après la recherche méthodique des provinces magmatiques principaux du monde, mes conclusions sur cette question. Il me suffit maintenant de remarquer, qu'il y a quelque chose de juste dans la supposition de M. STARKER sur ce point-là.

Revenant à la mafraïte N°. 18, il faut remarquer que cette roche ne contient pas d'olivine, mais au lieu de cela, de l'amphibole et de la biotite. Aussi y-a-t-il lieu de supposer que la relation de réaction (le „reaction relation" de BOWEN) n'a pu réussir ici, l'olivine étant dissimulée dans des minéraux bien stabiles. C'est probablement la teneur en gaz du magma (laquelle se manifeste dans la sodalite, l'haiiyne des tahitites, etc.), qui a contribué à la formation de l'amphibole et de la biotite au lieu de l'olivine et aussi à la formation de l'andésite N°. 44.

Etablissons encore quelques relations entre les roches de Tahiti et le N°. 45 :

b. La tahitite N°. 39 peut avoir été la liquide-mère du microgabbro N°. 11 :

	si	$2fe_2O_3+feo$	*mgo*	*c*	*al*	*na*	*ka*
$\frac{1\times N°.39+19\times N°.11}{20}$:	95	22.1	29.4	*23.8*	17.5	*5.5*	1.6
N°. 45 :	100	22.1	29.4	*24.4*	16.0	*6.5*	1.6

c. Le microgabbro N°. 11 peut être mis en relation aussi avec le camptonite N°. 27 :

	si	$2fe_2O_3+feo$	*mgo*	*c*	*al*	*na*	*ka*
$\frac{1\times N°.27+1\times N°.11}{2}$:	90	22.3	29.4	*24.1*	16.0	*5.5*	1.8
N°. 45 :	100	22.1	29.4	*24.4*	16.0	*6.5*	1.6

Ces deux roches ne diffèrent pas beaucoup du N°. 45 ; dans ce cas, la différenciation a probablement été arrêtée par le refroidissement causé par les dimensions de la chambre magmatique.

d. L'ankaramite basaltique N°. 53 a eu pour liquide-mère un magma luscladitique N°. 12 ; probablement le N°. 53 est redevenu liquide par diminution de pression, ou (en) par échauffement par en bas (voir mon article sur le Katmai, op. cit.)

	si	$2fe_2O_3+feo$	*mgo*	*c*	*al*	*na*	*ka*
$\frac{7\times N°.53+6\times N°.12}{13}$:	97	*23.6*	*27.6*	24.6	15.9	6.1	*2.2*
N°. 45 :	100	*22.1*	*29.4*	24.4	16.0	6.5	*1.6*

e. Le basanitoïde N°. 46 et la yamaskite péridotique N° 20 sont complémentaires l'un de l'autre :

	si	$2fe_2o_3 + feo$	*mgo*	*c*	*al*	*na*	*ka*
$\frac{4 \times N°.\ 20 + 3 \times N°.\ 46}{7}$:	90	22.0	*28.3*	24.7	16.0	6.5	*2.5*
N°. 45 :	100	22.1	*29.4*	24.4	16,0	6.5	*1.6*

f. La syénite néphélinique N°. 1, la microbérondrite N°. 19 et la mafraïte N°. 18 donnent ensemble le magme demandé :

	si	$2fe_2o_3 + feo$	*mgo*	*c*	*al*	*na*	*ka*
$\frac{2 \times N°.19 + N°.1 + 10 \times N°.18}{13}$:	93	23.0	27.8	24.0	16.1	6.6	2.5
N°. 45 :	100	22.1	29.4	24.4	16.0	6.5	1.6

On pourra citer encore plusieurs autres exemples.

Quand on considère toutes ces roches, il est bien difficile de s'imaginer qu'elles viennent toutes d'une seule chambre magmatique. J'ai fait allusion plusieurs fois à l'arrêt par refroidissement de la différenciation magmatique. Il est beaucoup plus en accord avec les phénomènes observés d'accepter l'existence de plusieurs chambres magmatiques, la différenciation magmatique dans chaque chambre ayant son histoire propre, plus ou moins longue, dépendant du moment auquel le magme trouvait son chemin vers la surface ou auquel il était arrêté par le refroidissement à cause de ses dimensions. Mais quand il y a plusieurs chambres magmatiques, le fait, indiqué plus haut, sur la teneur en fer, en magnésie et en alcalies gagne en importance. Les magmas n'ont pas eu exactement la même composition ; entre ces magmas règne une relation, constituée généralement dans les *roches* éruptives. En outre, la composition du basalte N°. 45 correspond à celle d'une amphibole et il est très remarquable, que la composition de celle-ci diffère très peu de la composition de l'amphibole hypothétique, que j'ai signalé dans mon article sur l'Etna ; c'est à dire :

	si	$c + fm$	*al*	*alk*
Etna	78 à 140	74.5	16.9	8.6
Tahiti	100 [1])	75.9	16.0	8.1

A cause de toutes ces relations régulières et aussi comme conséquence de mes observations encore inédites sur les roches de l'Etna, du Katmai et d'autres régions, je prétends que les magmes des roches de Tahiti proviennent de la refusion d'une roche amphibolique ; les roches les plus profondes donnant les magmas les plus magnésiens. La cause de cette refusion doit être cherchée dans une diminution de pression, aussi bien que dans un échauffement par en bas.

L'origine de cette couche amphibolique que nous trouverons pure dans

[1]) Les autres magmas de Tahiti ont $Si < 100$. Cela peut s expliquer de deux manières L'une de ces manières c'est que la cristallisation de l'olivine, qui est présente en N°. 45, enrichissait la liquide-mère en silice. L'autre est exposée dans le texte de la présente note.

le Pacifique central et feldspathiseé dans plusieurs régions des Continents, date probablement de la solidification originelle de la Terre entière ; la teneur en gaz des magmas, étant de beaucoup plus grande dans ces temps-là qu'aujourd'hui, ayant favorisé la formation d'amphibole. Nous assistons actuellement à une seconde phase du Volcanisme, commencée au moment de la solidification de l'écorce terrestre.

Cette couche amphibolique remplace la couche basaltique de Daly.

Il y a beaucoup de conclusions à faire ; mais je préfère accumuler d'abord plus de données. Seulement je ferai remarquer qu'il est probable qu'on trouvera dans les laves et spécialement dans les laves basiques, deux espèces d'enclusions homoeogènes : la première espèce, consistant en nodules à amphibole, provient de la roche originale et la deuxième espèce, consistant en nodules à olivine, provient de la roche refondue et recristallisée. Il est probable aussi qu'on trouvera des amphiboles en voie de fusion [1]).

Revenons à la pétrologie ordinaire des roches de Tahiti. J'ai déjà remarqué que les roches volcaniques montrent les phénomènes de la différenciation magmatique mieux que les roches intrusives et filoniennes. On voit, dans la figure 2, que la potasse se manifeste principalement dans la phase acide et surtout dans la phase ultra-acide, faits bien connus. On voit aussi que M. A. LACROIX a eu bien raison en établissant la famille des Tahitites ; elles forment une famille bien marquée à laquelle appartient aussi le N°. 35, qui a été nommée phonolite à haüyne par IDDINGS ; les phonolites et les trachytes appartiennent aux liquides ultérieures des magmas de Tahiti et ce n'est pas le cas avec le N°. 35. Les roches 3 et 35 sont probablement différenciées d'un magma de composition tahititique ; l'une des deux a un excès d'aluminium sur les tahitites, ce qui manque à l'autre Elles sont probablement les produits d'une chambre magmatique aux dimensions assez larges, de telle sorte que le refroidissement n'a pu animer la différenciation magmatique ; plus tard, le magma tahititique s'est différencié en deux phases plus ou moins riches en aluminium ; le magma tahititique étant-lui-même liquide-mère surmontant un gabbro dans la profondeur ou bien une pyroxénolite suivie d'une bérondrite [2]):

	si	$2fe_2o_3+feo$	*mgo*	*c*	*al*	*na*	*ka*
$\frac{8\times N^0.21+14\times N^0.17+7\times N^0.41}{29}$:	88	21.4	28.6	25.2	16.1	6.9	1.8
N°. 45 :	100	22.1	29.4	24.4	16.0	6.5	1.6

A la famille des tahitites appartient aussi la roche N°. 43 (nommée

[1]) M. A. LACROIX a trouvé dans une ordanchite de Madagascar des cristaux d'amphibole, montrant des phénomènes de fusion. cf. A. LACROIX-Minéralogie de Madagascar. T. III, p. 51.

[2]) La bérondrite N°. 17, on le voit dans la figure 3, a une position assez différente des autres bérondrites.

tautirite par IDDINGS) comme l'a déjà remarqué M. A. LACROIX et comme on le voit dans la figure.

D'après celle-là on peut établir une classification des roches volcaniques. La phase basique peut être divisée en deux parties, selon que les roches sont plus ou moins pauvres en sodium ; la phase acide peut être divisée également en deux parties, selon que ses roches sont plus ou moins riches en aluminium. On obtient alors le tableau suivant :

PHASE BASIQUE.

1. (Très pauvre en sodium)		2. (Moins pauvre en sodium)	
Ankaramite-océanite	(56)	Ankaratrite porphyrique	(57)
Ankaramite-océanite	(55)		
Ankaramite-basanitique	(53)	Basanitoïde holocristallin	(52)
Ankaramite-basanitique	(54)	Ankaratrite étinditique	(58)

Basanitoïde porphyrique (51)

PHASE ACIDE.

Basanitoïde porphyrique (49)

1. (Moins riche en aluminium)		2. (Plus riche en aluminium)	
Basanitoïde porphyrique	(48)	Andésite	(44)
Basanitoïde aphyrique	(47)		
Basanitoïde	(46)		
Tahitite	(35)	Tahitite	(39)
Tahitite	(41)	Tahitite	(38)
Tahitite	(42)	Tahitite	(43)

PHASE ULTÉRIEURE

Trachyte phonolitique	(31)	Trachyte phonolitique	(32)
		Trachyte phonolitique	(33)
		Phonolite	(34)

Tinguaïte (36)

Je ne veux pas insister pour l'instant sur une classification des roches intrusives et filoniennes de Tahiti. Il entre, avec ces roches, un élément étranger à la composition chimique ; c'est le refroidissement dû aux dimensions des chambres magmatiques. Je crois que c'est un des éléments qui causent tant de difficultés à toute classification. D'ailleurs, on peut voir, dans les figures, avec quelles roches volcaniques elles correspondent le mieux.

Bandoeng, 7 août 1929.

Chemistry. — *The Formation of cis-cis-Muconic Acid from Benzo-o-Quinone with Peracetic Acid, a Simple Transition from an Aromatic to and Aliphatic Compound.* By Prof. J. BÖESEKEN and Mr. G. SLOOFF.

(Communicated at the meeting of October 26, 1929).

In a foregoing communication [1]) we gave a short survey of the action of peracetic acid on the triple bond. It was stated there that this bond was much more slowly attacked than a double bond, the result being a separation into two parts, in such a way that the original unsaturated C-atoms had passed into carboxyl groups. As it was supposed that one of the intermediat products would be an o-diketo compound, which would further be oxidized to the two carboxyl groups, some of these diketones were exposed to the action of peracetic acids; among these were *stearoxylic acid* and *o-naphtoquinone*. In fact the first acid quickly gave nonanic acid and azelaic acid, the second substance gave cis-cinnamon-ortho-carbonic acid. In this latter oxidation the double bond next to the carboxyl group is left intact, at least it is attacked only very slowly, as might be expected from such a double bond [2]). This oxidation led us to examine the behaviour of peracetic acid with respect to benzo-o-quinone, as we might expect *muconic acid* here, a substance in which *both* the double bonds are each situated beside a carboxyl group, and which would, accordingly, be only slowly attacked by the oxidising agent.

This expectation proved to be true: to a very satisfactory amount the o-quinone passed into an acid, the analysis values of which left no room for doubt of the constitution $C_4H_4(COOH)_2$. The melting point 195° comes nearest to a muconic acid, of which the methyl ester has been obtained by H. A. FARMER [3]) in the decomposition of the $\beta\beta'$ dibrom adipic acid esters with $NaOCH_3$ or pyridine (by the side of the methyl ester of the high melting ($\pm$ 300°) muconic acid, which has already long been known). Of this latter acid it was shown by R. BEHREND [4]) that on careful oxidation with alkalic $KMnO_4$ besides a little mucic acid exclusively uvic acid (no anti-tartaric acid) was obtained. Hence he considered the trans-trans-structure proved for this muconic acid. At the time this conclusion was subject to doubt: however, now that it has been shown

[1]) These Proc. **32**, 377 (1929).
[2]) Recueil **45**, 838 (1926).
[3]) Journ. Chem. Soc. **123**, (1923).
[4]) B **49**, 999 (1916), compare also Annalen **418**, 294 (1919).

that on oxidation of cyclenes with alk. $KMnO_4$ exclusively cis diols are formed[1]), the certainty has become much greater.

This is also the case now with the genetic connection of the mucic acid and the high melting muconic acid.

As regards the muconic acid obtained by us, on the strength of its synthesis from o-quinone, in which the double bonds are left intact, this will undoubtedly be the cis-cis-muconic acid, which is symbolized by the sub-joined figures:

If on closer investigation the identity should appear of our acid with that of FARMER (loc. cit.), then the configuration would be established also of this. In the case that they are not identical, the same holds good, as there are no more than three isomeric muconic acids possible FARMER's acid would then be the cis-trans acid.

These conclusions must be confirmed by further investigation: the behaviour on oxidation with $KMnO_4$ etc. In anticipation of this we may say already now that the presence of trans-trans-muconic acid in the urine of rabbits or dogs fed on benzene, cannot be due to direct oxidation of this hydrocarbon in the animal body, as then the cis-cis-isomer should have formed.

If it should really be true that on addition of benzene to the food, trans-trans acid was separated by the test animals, this is more likely the consequence of a *reduction* and dehydration e.g. of galactose, as the grouping of the atoms of this sugar and the closely allied mucic acid renders the formation of trans-trans-muconic acid possible.

Experimental part.

The *o-quinone* is prepared as follows. One gramme of silver oxide, which had been obtained by precipitation of $AgNO_3$ with a small excess of KOH, and had then been washed with water, pure dry acetone and dry ether, and carefully dried over P_2O_5, was suspended in a shaking cylinder with 15 ccm absolute ether, to which 1 gr. anhydrous Na_2SO_4 had been added, then 0.250 gr. pyrocatechol was added, after which it was shaken vigorously for 4 minutes. Then the solution of the quinone was filtered through a folding filter, on which anhydrous Na_2SO_4 had been put. Now the ether is quickly evaporated to a volume of

[1]) Recueil **47**, 683 (1928).

four ccm, and then cooled in ice. The o-quinone then separates in well-formed red crystals from which the mother liquid is poured off.

Then a few drops of 20 % acetic acid solution of peracetic acid are added to this o-quinone; the quinone goes in solution with slight development of heat, assuming a light-yellow or yellow-brown colour. After some time colourless crystals are deposited, rhombic, sometimes elongated to needles. From 4 samples of 250 mg. pyrocatechol, 130 mg. were obtained. Melting point = 195°.

0.1089 gr. yielded 0.2014 gr. CO_2 and 0.0416 gr. H_2O.

0.0391 gr. neutralize 5.33 ccm. 0.1029 *n*-barite.

Found C = 50.4 %, H = 4.24 % Mol. weight (2 basic acid) = 142.6.

Calculated ($C_6H_6O_4$), C = 50.7 %, H = 42 % Mol. weight 142.

For the *hydromuconic acid* a melting point of 195° is given in the literature, but the hydrogen content of this = 5.5 %, besides it is not to be explained how this acid could have been formed here. FARMER gives for the isomer found by him the melting point 187°.

Delft, Sept. 1929.

Physics. — *Methods and apparatus used in the cryogenic Laboratory.* XXIII. *A horizontal cryostat for the measurement of magnetic susceptibilities at low temperatures.* By E. C. WIERSMA and H. R. WOLTJER. (Communication N°. 201*c* of the Physical Laboratory at Leiden). (Communicated by Prof. W. J. DE HAAS).

(Communicated at the meeting of October 26, 1929).

§ 1. *Introduction.* One of the methods generally used for the measurement of magnetic susceptibilities depends on the determination of the forces exerted on the substance by an inhomogeneous magnetic field. In previous experiments the arrangement was such that these forces were directed vertically. The substance under investigation was attached to a vertical rod. The weight of the system formed by the rod and the substance was balanced by floats immersed in mercury. The substance was placed above or below the centre of an electromagnet and the magnetic attraction or repulsion acting along the vertical is measured either by additional weights [1]) on the floating system or by the interaction of suitably arranged coils [2]). This system, however, would drift away under the influence of the slightest asymmetry. This difficulty was overcome by guiding the system in its up and down motion by means of two flat spiral springs. The unavoidable stiffness in vertical direction of these springs destined to prevent horizontal displacements limited the sensitivity. The greatest accuracy obtained by us was about 7 mg. As an increase in the sensitivity was needed, especially for work on the susceptibility of oxygen at high densities, an attempt was made to adapt for low temperature work the method used by WEISS and his collaborators [3]), in which the force is exerted in a horizontal direction.

Owing to the difficulty of constructing a cryostat for liquid cooling agents for the purpose mentioned, it was decided to make use of cold gases. By this method also the difficulty of the changing hydrostatic pressure of the liquid during the measurements is obviated. [4])

This method of cooling has the additional advantage that it can be used over a wide temperature range, down to nearly 20° K. if cold hydrogen gas is the cooling agent and moreover for those temperatures where no liquids are available. The range between 20° and 65° K.

[1]) E. OOSTERHUIS, Leiden Comm. N°. 139*b*.

[2]) H. KAMERLINGH ONNES and A. PERRIER, Leiden Comm. N°. 139a.

[3]) P. WEISS and G. FOËX, J. Phys. **5** (1911) pp. 1, 275, 744, 895; G. FOËX and R. FORRER, J. Phys. **7**, (1926), p. 180.

[4]) This difficulty has since been overcome in another manner; see W. J. DE HAAS, E. C. WIERSMA and W. CAPEL, Leiden, Comm. N°. 201*b*.

(boiling point of hydrogen and freezing point of nitrogen) is of particular importance, as it is in this range that certain anhydrous chlorides undergo the transition to the condition which exists at liquid hydrogen temperatures when the magnetic susceptibility is dependent on the field strength.[1])

Cryostats working on this principle satisfying a high demand for constancy of temperature have been in use for some time.[2])

In the present case it was necessary that the parts lying between the poles of the electromagnet should be of small dimensions. On the other hand, however, a temperature constant to 0.1 to 0.05° was sufficient.

An apparatus has now been built making use of the method of WEISS in conjunction with a vapour cryostat for maintaining the substance under investigation at a low temperature. As this apparatus has been found to work satisfactorily a short description of it is given.

§ 2. *General Considerations.* The small tube containing the substance under investigation was fastened to one end — the right hand end — of a glass tube — the carrier — hung horizontally from two pairs of wires, each pair in the form of a V. (fig. 1). The left hand end carried a coil. At either end of this coil, coaxial with but free from it, were placed two fixed coils. By regulating strength of the current flowing in the correct direction through these coils it was possible to balance the force exerted on the substance by the field of an electromagnet, and thus to determine the susceptibility.

By means of a sensitive arrangement of mirrors[3]) it was possible to determine when the carrier was in its null position. To protect this device, the carrier must not be subjected to shocks or large displacements. For this reason the coils in series with a set of rheostats W_2, W_3, W_4, were connected in parallel with the electromagnet, and this system again in series with a further rheostat W_1 (fig. 2). The resistance of W_2 was first so chosen that for weak magnetic fields there was approximate equilibrium. The current strength was then increased with the rheostat W_1 when the equilibrium was but slightly disturbed[4]) and could be restored at any moment with the resistances W_3 and W_4, and finally adjusted when the current through the electromagnet had reached the desired value.

The two pairs of wires supporting the carrier were used as leads for

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[1]) H. R. WOLTJER, Leiden, Comm. N°. 173*b*.
H. R. WOLTJER and E. C. WIERSMA, Leiden, Comm. N°. 201*a*.

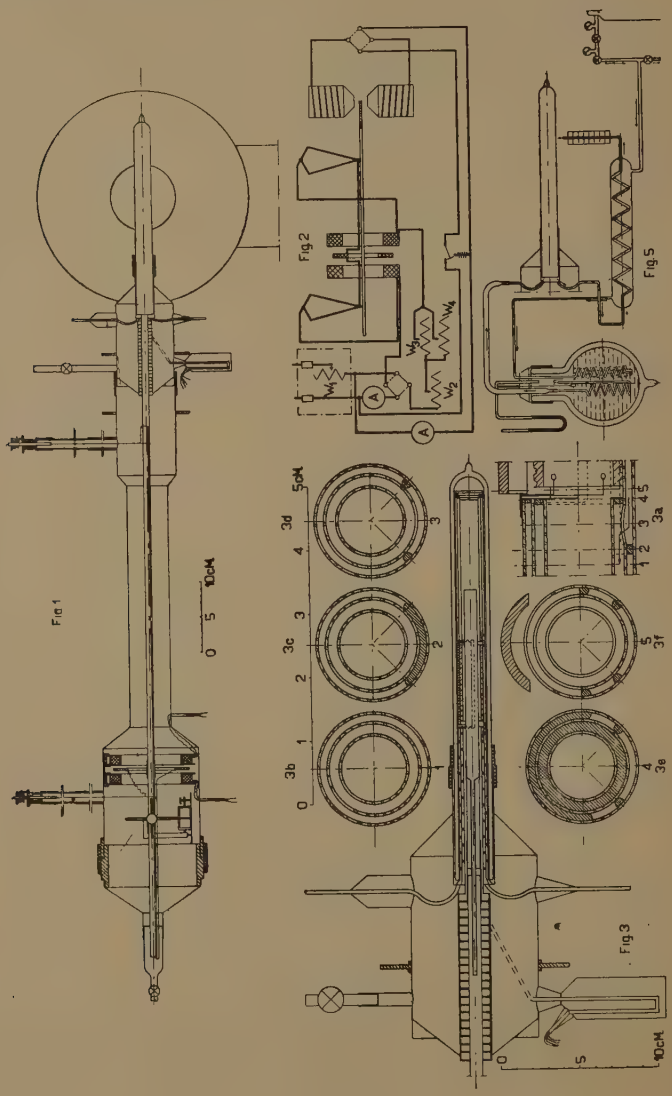
[2]) C. A. CROMMELIN, Leiden, Comm. Suppl. N°. 45, § 11.

[3]) This is a copy of the arrangement described by KOPP, see W. KOPP, Diss., Zürich 1919.

[4]) The force between the coils is proportional to the squared current strength and this holds sufficiently well also for the force exerted by the field on the (para-magnetic) substances since, for the current used by the magnet, the field is approximately proportional to the current strength. Naturally the accuracy could be increased by measuring in the region of the saturation field, but the magnet would then become much too hot.

the current through the movable coil. The length of the wires could be adjusted by screws running in packing tubes until the carrier hung freely in the cryostat.

The whole system of carrier, supporting wires and mirrors was contained in an air tight enclosure, to the right hand end of which the cryostat was attached.



§ 3. *The Cryostat.* (Fig. 3). The most important part of this consists of three concentric new silver tubes [1]) (Fig. 4) [2]). Cold vapour from liquified gas (air or hydrogen) is circulated through the sheaths between the tubes, called the inner and outer circulation spaces. The cold gas has been given a spiral motion in the left hand part of the system. For this reason a spiral of copper wire, about 1.5 mm thick was soldered unto the inner tube (internal diameter 10 mm, external diameter 11 mm). The whole was turned on a lathe to exactly 14 mm ext. diameter, and placed inside a new silver tube of 14 mm ext. diameter. In a similar manner a copper wire about 1 mm thick had been soldered unto this second tube and the whole turned down to 17 mms.

Fig. 4.

These tubes were placed inside a vacuum glass of 19 mm. internal and 24 mm external diameter. The length of the tubes was calculated for the symmetrical type of investigation tubes. [3]) In these the form of the glass is symmetrical and the plane of symmetry passes through the centre of the field of the pole pieces. The forces on the glass are eliminated but the length is increased. After the experimental tube had been suitably connected to the carrier, small covers were soldered to the right hand ends of the inner and outer new silver tubes. The circulation space was thus closed off from that containing the substance under investigation.

It was feared that there might be considerable flow of heat from the hot to the cold spaces, causing strong convection currents which would disturb the carrier. This difficulty was overcome in two ways. Firstly, the dimensions were made such that the space round the carrier was as small as did not interfere with its free movement. The internal diameter

[1]) It would have been preferable to have used a material such as copper for the right hand part to obtain a more even temperature distribution. This however would have offered great technical difficulties. New silver also had the advantage that accurately worked tubes of suitable sizes were obtainable An important point in the choice of the material is the rate of conduction of cold from the gas, but little is known on this subject.

[2]) The small projections, to be seen in the photograph above the middle tube and the under part of the right hand tube, are due to the strings used to fasten the tubes on the paper for photography.

[3]) H. KAMERLINGH ONNES and A. PERRIER, l.c.

of the inner tube was 10 mms, the external diameter of the experimental tube 8 mms. Secondly, the space between the cold and hot parts was occupied for a length of about 11 cms by a system of baffles in the form of fibre rings placed two or three mms apart. The expectation that convection currents would lose their kinetic energy through the formation of vortices between these baffles was later found to be justifiable (see below.)

It was of great importance that the temperature of the substance under investigation should be as much identical as possible with that indicated by the platinum resistance thermometer. The porcelain former, on which the resistance wire was wound, closely surrounded the investigation tube, having the same internal diameter as the inner new silver tube. It had an external diameter slightly less than the internal diameter of the middle tube.

Space for the thermometer was obtained by cutting away the new-silver tubes over the length of the former. A complete section was cut out from the inner tube so that this actually consisted of two parts. Half sections were cut from the other two tubes. A part of the outer circulation space, shut off by a closed loop of copperwire fixed to the middle tube, was reserved (see fig. 4) for the gas flowing from the right to the left hand part of the inner space.

The path of the cold gas through this part of the circulation space can be seen from the cross sections shown in figs. *3a—3f* taken in conjunction with fig. 4. The thermometer was covered with a copper lid in the form of a half cylindermantle to ensure uniformity of temperature. The leads for the resistance thermometer ran between the vacuum glass and the outer new silver tube and then through a separate enclosure cooled by liquid air. Heat conduction along the leads was thereby prevented.

§ 4. *Cooling.* In the present research, for which this cryostat was primarily designed and in which no temperatures below 154° K. were needed, cold air was used as cooling agent. Air from a cylinder (Fig. 5) was bubbled through liquid air in a large reservoir (5 litres capacity) in order to cause evaporation. The cold vapour was further cooled by liquid air and then led through a DEWAR tube into the cryostat. As the air leaving the cryostat was still cold, it was passed through the spiral of a glass condenser. In this way it served to cool the air from the cylinder, which was passed through the jacket of the condenser before reaching the 5 Liter reservoir. This somewhat primitive regeneration was found to work efficiently, the used air being brought up to room temperature. No moisture condensed on the flowmeter, unless at very high velocities.

The rough adjustment of the rate of flow was made with the reducing valve of the pressure cylinder and the fine adjustment with a valve in a parallel tube. In practice the rate of flow was not used for the adjust-

ment as much as the pressure in the large reservoir. This was read on a long, open oil manometer, and was regulated according to the directions of the observer controlling the temperature. So long as there was no obstruction, the pressure corresponded unequivocally to the velocity but could be read more accurately. No trouble with obstructions was experienced if care was taken, when the apparatus was not in use, to keep the 5 Litre reservoir filled with liquid air and the exit of the circulation space closed. The latter was thus kept perfectly dry.

§ 5. *Details.* The temperature difference between the surroundings of the cryostat and the internal cold part was in the first place dependent on the rate of flow of the gas, and could be considered proportional to it so long as the temperature was not too low. The change in temperature of the liquid air was of secondary importance. Table I gives the relation between the absolute temperature T reduced on $T = 291°$ K

TABLE I.

v	T
100 L/hour	254° K.
200	216
300	179
400	150
500	139
600	134

as temperature of the surroundings and the rate of flow v to a first approximation. From this it was also possible to obtain a qualitative idea of the thermal insulation. Thus, to maintain a temperature difference of 60 degrees a rate of flow of 160 Litres (NPT) per hour was necessary. Assuming that the air entered the cryostat as cold as possible (say 83° K) left the condenser at room temperature (say 291° K), and warmed up at constant pressure ($c_p = 0{,}25$), then 160 L $= 160 \times 1{,}3 \times 10^3$ gms gained about

$$160 \times 1{,}3 \times 0{,}25 \times (291 - 83) = 1{,}08 \times 10^4 \text{ cals/hour.}$$

There was therefore a thermal leak of $\frac{1.08 \times 10^4}{60} = 180$ cal/hour for one degree temperature difference.

Actually the leak was smaller on account of the higher initial, and the lower final temperature of the cold air. About a 100 cals. are necessary to vaporise 1 gm of air and bring it to room temperature

at constant pressure. With a leak similar to that discussed above, $180 \times 208/100 = 374$ gm or 467 cm^3 of air at 83° K must be vaporised and brought to room temperature per hour. The level of liquid air in a cryostat of cross section 47 cm^2 and having the same insulation as the vapour cryostat would therefore fall at ten $^{cms}/_{hour}$. Bearing in mind the open connection between the cold and the hot spaces this result is not un-satisfactory.

Table I could be used also as an aid in adjusting the temperature.

Fig. 6*a* and 6*b* show the variation of the temperature during an experiment on two occasions.

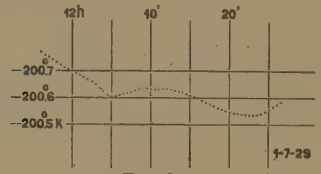


Fig. 6*a*.

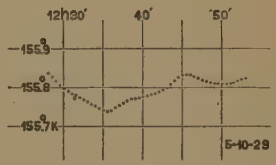


Fig. 6*b*.

With regard to the accuracy of the magnetic measurements the following remarks may be made. With the aid of the mirror system and a telescope the displacement was magnified rather less than 500 times, and since mms could be read with sufficient accuracy in the telescope, it was possible to reproduce the equilibrium position to about 0,002 mms by regulating the current through the coils. The corresponding force was naturally dependent to some extent on the weight of the investigation tube attached to the carrier, but was of the order of 0,2 mgm. As the forces are generally of the order of 1 gr. or more, the magnetic attraction could be compensated with an accuracy of at least 2 in 10.000. The experimental errors in the *measurement* of the force is a given field could, however, certainly amount to 6 $^0/_{00}$, since the ammeters in the circuits of the coils and the magnet were not certain to more than 1,5 $^0/_{00}$ and this error is squared in both readings. [1])

§ 6. *Testing of the Apparatus.* The extent to which the temperature of the substance under investigation was the same as that registered by the thermometer, was shown in the following ways. Firstly, when the temperature was allowed to pass through a minimum, the susceptibility passed through a maximum with a very short time lag. Secondly, control measurements were made with a tube containing gadolinum sulphate [2]) on different days, the temperatures being repeated and measurements made for rising and falling values.

The value of the current i (in milliamps) through the coils is proportional

[1]) As the magnetic field is approximately proportional to the current. See note 4, p. 1047.

[2]) Used previously by H. KAMERLINGH ONNES and E. OOSTERHUIS, Leiden, Comm. Nº. 129*b*, § 6.

to the susceptibility. In table II i^2 has been corrected for ammeter errors, for the empty carrier and for the anion, although these corrections do not amount to more than a few $^0/_{00}$.

TABLE II.

Date	$10^{-2}\ T$	$10^{-2}\ i^2$	$10^{-4}\ i^2T$	i^2T (calc.—obs.) in $^0/_{00}$
27 June '29	287.9° K.	1457	4195	+ 2
„ „ „	233.2	1781	4153	— 8
29 „ „	288.7	1456	4203	+ 4
1 July „	227.3	1840	4182	— 1
	200.6	2075	4162	— 6
	170.7	2466	4209	+ 4
2 „ „	289.0	1452	4196	+ 2
	160.3	2604	4174	— 3
5 „ „	170.2	2474	4211	+ 6
6 „ „	289.4	1446	4183	+ 1

The last column gives the deviation from the mean of i^2T in $^0/_{00}$. As the uncertainty in the magnetic measurements can amount to some parts in thousand, there is no reason to doubt the accuracy of the temperature to 0.1° or 0.05° degree.

Calculation by least squares gave the most probable value of θ and C in the equation $i^2(T-\theta) = C$ as 0.4° and 4180 with probable errors of $\pm$ 1.5° and $\pm$ 30 respectively.

These determinations may also be used to calibrate the apparatus, The value of the force in mgms/milliamp^2 was found to be 6.75×10^{-3} from the following data: weight of cylinder of gadolinium sulphate, 2.897 gms., length 8.74 cms, field strengths at the ends of the cylinder (magnet current 15 amps.) 9179 and 842 gauss, Curie constant 2.016×10^{-2}.

§ 7. *Summary.* A cryo-magnetic apparatus has been built making use of the method of WEISS and his collaborators for the measurement of magnetic susceptibilities in conjunction with a vapour cryostat for maintaining the substance under investigation at a low temperature.

We wish to express our thanks to Prof. W. J. DE HAAS for the opportunity of carrying out this research, to Mr. L. OUWERKERK, technician 1st class, for the able construction of the cryostat and to Mr. C. W. COPPOOLSE, assistant of the Physical Laboratory, for his valuable help in the measurements.

Endocrinology. — *Over de verandering van het Veerenkleed bij Hoenders door Menformon,* door J. FREUD, S. E. DE JONGH en ERNST LAQUEUR. (Communicated by Prof. B. BROUWER.)

(Communicated at the meeting of October 26, 1929).

Over de verschillende werkingen van *Menformon* [1]) handelen meerdere mededeelingen [2]), die van uit ons laboratorium bewerkt en waarvan 2 in deze verslagen gepubliceerd werden; tot nu toe werden door ons echter alleen verschijnselen bij zoogdieren beschreven [3]). Wij hebben daarom nu proeven verricht op *vogels*. Juist bij deze toch heeft een bepaald geslachtskenmerk, n.l. het *veerenkleed* veel belangstelling gewekt; terwijl veel schrijvers het vrouwelijke veerenkleed met name bij hoenders als afhankelijk van een ovariaalhormoon beschouwen, wordt dit door anderen tot op dit oogenblik betwijfeld. De volgende onderzoekingen toonen aan, dat inderdaad een vrouwelijk hormoon en wel Menformon geheel of gedeeltelijk aansprakelijk is voor het ontstaan van het vrouwelijke type van veeren bij hoenders.

De proeven werden verricht met preparaten, die door Mej. DINGEMANSE waren bereid en een zuiverheidsgraad bereikten, waarbij de eenheid in ongeveer 0.01 mg. droge stof aanwezig was. Hiermede werden zoowel gecastreerde hennen als gecastreerde en normale hanen behandeld.

Het veerenkleed van de hennen en hanen van het gebruikte hoenderras: Patrijs Leghorn (leghorn doré), is zeer karakteristiek en duidelijk te onderscheiden. Wordt een hen gecastreerd, dan krijgt ze bij de volgende veerenwisseling (na uittrekken der veeren of na ruien) een z.g. neutraal veerenkleed, dus precies als een mannelijke castraat. Dit type van veeren gelijkt zeer veel op dat van den normalen ongecastreerden haan, is alleen nog rijker ontwikkeld.

Wij willen hier geen nauwkeurige beschrijving van het veerenkleed geven, maar beperken ons tot drie kenmerken, die in het bijzonder onze aandacht hadden en die ook volgens kenners op dit gebied karakteristiek zijn.

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[1]) Onder Menformon moet worden verstaan een vrouwelijk geslachtshormoon in een oplossing van zoodanige zuiverheid, dat in minder dan 0.1 mg de z.g. muizeneenheid aanwezig is. Deze is die hoeveelheid, welke bronst bij de gecastreerde vrouwelijke muis teweeg brengt. Aan deze definitie hebben wij niets veranderd, ofschoon wij reeds sinds lang een ongeveer 1000 maal zoo grooten graad van zuiverheid ($\pm$ 0.1 γ pro M. E.) hebben bereikt.

[2]) Versl. Kon. Akad. Amsterdam, Dl. **34**, N°. 10, 1925 en Dl. **35** N°. 2, D. m. Wochenschr. 1926, 1927, 1928, N. T. v. Geneesk. 1927, 1928, 1929 en Kl. Wochenschr. 1927, 1928, enz.

[3]) RIDDLE (An. Journ. Physiol. **87**, 97, 1928) beschreef een groeitemperenden invloed van vrouwelijk hormoon op den eileider van duiven, tevens nam hij remming van testikel en ovarium waar.

1. Bij hennen bruine borstveeren, tegenover zwarte bij hanen en ook bij mannelijke en vrouwelijke castraten.

2. Bij hennen het ontbreken van pronkveeren bij het zadel, terwijl hier bij hanen en castraten mooie, goudgele, lange spitse veeren aanwezig zijn.

3. Bij hennen wederom het ontbreken van karakteristieke dekveeren aan de vleugels tegenover zwarte veeren bij hanen en castraten.

Wij hebben nu kunnen aantoonen, dat Menformon in voldoende hoeveelheid twee maal daags ingespoten, in staat is bij gecastreerde kippen en hanen, zoowel als bij normale (ongecastreerde) hanen een veerenkleed te doen ontstaan, dat ook wanneer het niet steeds geheel vrouwelijk is, zich toch zeer duidelijk in vrouwelijke richting onderscheidt van het neutrale type (hanentype). D.w.z. de borstveeren bevatten meer bruin, fraaie dek- en zadelveeren ontbreken. Deze veranderingen komen niet tot stand door processen in de aanwezige veeren; komen er echter (na plukken of rui) tijdens Menformon-invloed nieuwe veeren, dan zijn deze „vrouwelijk" in bovengenoemden zin.

Zijn de gegeven hoeveelheden niet groot genoeg, om de volle ontwikkeling van het vrouwelijke veerenkleed te verkrijgen, dan ontstaan (ten minste bij hanen) mengvormen, waarbij het bruine pigment niet voor het geheele borstveerenkleed voldoende schijnt en men bruine dwarsstrepen over de overigens zwarte veeren ziet ontstaan.

Zooals vroeger werd meegedeeld, kan men ook zekere hoeveelheden Menformon uit mannenurine extraheeren (ongeveer 50—200 eenheden per Liter).

Niet zonder belang is het, dat dit Menformon uit mannenurine bereid principieel dezelfde resultaten oplevert, n.l. verandering van het neutrale veerenkleed in de richting van het vrouwelijke type. Interessant is, dat zulk Menformon uit mannenurine bereid, tegelijk ook nog *mannelijk* hormoon bevat. Dit hormoon veroorzaakt, zooals voor kort FUNK, KOCH c.s. aangetoond hebben, groei van den kam. De behandeling van gecastreerde hennen met dit Menformon bewerkt nu tevens, dat de kam iets groeit, zich opricht en rood wordt, d.w.z. een mannelijk karakter aanneemt. Op het mannelijke hormoon willen wij hier echter niet verder ingaan.

Proef 1.

4 bruine leghorn hanen gecastreerd 1, VII, 29; 4 bruine leghorn hennen gecastreerd 3, VII, 29.

De operatie is gelukt bij 3 hanen, 2 hennen. Leeftijd van de dieren ± 4 maanden. Alle 5 dieren werden links kaal geplukt, hoofdzakelijk aan de borst.

Menformonbehandeling begonnen 6, VII, bij de hanen N°. 17 en 22 en de hen N°. 26, onbehandeld bleven de hen N°. 35 en de haan N°. 25.

Het preparaat, een extract uit urine van vrouwen, bevat 1 M. E. in 0.01 mg.

Oplossing in olie 100 M. E. in 1 cc.

Dosis van 6. VII.—29. VII 2 maal daags ½ cc., van 29. VII.—12. VIII. 's morgens 1 en 's avonds ½ cc., dus in de eerste drie weken 100 M.E. per dag, daarna 150 M.E. p. d

Resultaat : bij het einde van de proef hebben aan den geplukten kant :

gecastr. hen 26 (Menf.) nieuwe bruine veeren met weinig zwarte vlekken en geen verandering aan het zadel ;

gecastr. hen 35 (onbehandeld) nieuwe zwarte veeren en goudgele zadelveeren (op beide zijden) ;

gecastr. haan 17 (Menf.) nieuwe bruin gevlekte zwarte veeren ;

gecastr. haan 22 (Menf.) nieuwe bruin gevlekte zwarte veeren ;

gecastr. haan 25 (onbehandeld) nieuwe zwarte veeren.

Uit dit resultaat volgt :

1. Dat de castratie van de hennen goed gelukt is, anders zou geen neiging tot zwartkleuring bestaan.

2. Dat het Menformon in staat is het vrouwelijke karakter van de veeren in stand te houden, resp. bij mannelijke castraten te voorschijn te roepen.

3. Dat dit effect misschien gemakkelijker te verkrijgen is bij vrouwelijke dan bij mannelijke castraten.

Voortzetting, resp. omkeering van de proef :

De contrôledieren van de vorige proef werden nu met Menformon in olie behandeld en de proefdieren uit de vorige proef alleen met olie. Haan 25 stierf 11. VIII., bij de sectie vond men geen resten van de geslachtsklieren.

De nog levende 4 dieren werden in het midden van de borst kaal geplukt.

Behandeling. Alleen hen N°. 35 werd met Menformon behandeld, dezelfde dosis van hetzelfde preparaat als bij de vorige proef (dus 's morgens 1 en 's avonds ½ cc.).

Duur van de proef 12. VIII—8. IX.

Resultaat : Bij het *einde* van de proef hebben op het geplukte deel van de borst :

gecastr. hen 35 (Menf.) bruine veeren

gecastr. hen 26 (olie) sterk zwart gevlekte bruine veeren (nawerking)

gecastr. haan 17 (olie) zwarte veeren } de bruingevlekte veeren van deze dieren in de vorige proef schreven wij dus met recht aan Menformon toe.

gecastr. haan 22 (olie) zwarte veeren }

Over de nawerking spreken wij nog nader.

Proef 2.

16 jonge hoenders, die 1 en 2. VI. 29 uit de kunstmatig uitgebroede eieren zijn gekomen en sedert 9—12. VII 29 zijn gecastreerd, worden in drie groepen verdeeld.

1ste groep : 4 hanen (36, 44, 34, 74) en 4 hennen (39, 32, 42, 46) krijgen Menformon, hetzelfde preparaat als boven.

2de groep : 2 hanen (38, 40) en 2 hennen (43, 31) krijgen olie.

3de groep : 2 hanen (30, 75) en 2 hennen (27, 29) blijven onbehandeld.

Alle dieren worden bij het begin van de proef links voor geplukt. Daar sinds de castratie niet geruid was, bezaten de gecastreerde hennen nog haar bruine veeren, terwijl ook de hanen door hun jeugd nog niet geheel zwarte veeren hadden, maar zwarte veeren met min of meer bruine vlekken.

Behandeling: *Menformon* 18. VII—26. VIII. voor de hennen en tot 8. IX. voor de hanen; *olie* 1. VIII—26. VIII. voor de hennen en tot 8. IX. voor de hanen.

Dosis van 18. VII—29. VII. 2 maal daags ¼ cc., d.i. 25 M.E., daarna 2 maal daags ½ cc. (dus eerst 50 M.E. en dan 100 M.E. p. d.).

Resultaat na het ophouden der inspuiting, geplukte zijde tegenover ongeplukte zijde:

1. Bij de Menformon hanen veel meer bruin;
   Bij de Menformon hennen in gelijke mate bruin.
2. Bij de contrôle hanen, zoowel als hennen (en wel de met olie behandelde als de onbehandelde) zijn de veeren veel zwarter. (De dieren zijn hun jeugdveerenkleed inmiddels blijkbaar ontgroeid.)

Verschillen in de diepte van het zwart van een geheele zijde komen daardoor tot stand, dat zich zwarte of nog zwart-bruin gevlekte jeugdveeren ontwikkelen, al naar den leeftijd van het dier.

Speciaal dient vermeld te worden, dat de goudgele spitse *zadelveeren* bij alle 8 Menformondieren *uitbleven*, terwijl zij bij de meeste contrôledieren inmiddels waren ontstaan.

Gedurende de proef waren vooral de hanen (bij wie de inspuitingen langer werden voortgezet) spontaan aan het ruien gegaan. Dit stoort onze resultaten geenszins; weliswaar werd door het nieuw verschijnen der henachtige veeren aan den niet geplukten kant het contrast links tegenover rechts minder duidelijk, maar de tegenstelling van behandelde dieren en contrôles als geheel werd des te sprekender.

Proef 3.

2 ongecastreerde hanen (60, 68) uit hetzelfde broedsel als van 2, worden beide met Menformon behandeld en wel met hetzelfde preparaat, dat in proef 1 en 2 is gebruikt.

Dosis: 's morgens 1 cc., 's avonds ½ cc., dus dagelijks 150 M.E. Deze haantjes waren goed ontwikkeld en deden kraaipogingen bij het begin van de proef, ook nog gedurende de eerste 2 tot 3 dagen van de behandeling, daarna niet meer.

Zij werden van 19/VIII tot 19/IX ingespoten.

De eerste karakteristieke verschijnselen vertoonden zich bij deze dieren op 8. IX. Bij het einde van de proef, ongeveer een halve week later, waren over het algemeen duidelijk vrouwelijke veeren te zien, evenwel nog hier en daar met zwarte vlekken. Deze proef bewijst, dat ook bij aanwezigheid van de tegengestelde gonade de werking van het vrouwelijke hormoon op de veeren ongestoord zijn gang gaat. Weliswaar is het feit, dat de hanen zich niet als zoodanig gedragen en ook niet meer kraaien een aanwijzing, dat de eigen mannelijke gonade vermoedelijk beschadigd werd (*antimasculine* werking [1]). Maar op de kleur van het veerenkleed heeft de mannelijke gonade toch geen invloed, daar, zooals wij reeds gezegd hebben, de castratie het veerenkleed van hanen slechts in quantitatieven zin beïnvloedt.

[1]) D. m. Wochenschr. 1926 N⁰. 30 en Ned. T. v. Gen. 1928, I, 24; Zeitschr. f. d. Ges. exp. Mediz. 1929, Bd. 68. Hfl. ¼.

Nawerking.

Wanneer men het gedrag van de met Menformon behandelde dieren na het einde van de behandeling verder bestudeert, doordat men van tijd tot tijd veeren plukt, of doordat, zooals dat bij ons ook het geval was, de dieren intusschen spontaan ruien, ziet men, dat de werking van de behandeling niet alleen *gedurende* de inspuiting waarneembaar is, maar dat een effect nog ongeveer 3 weken *daarna* te vervolgen is; ja men krijgt zelfs den indruk, dat vele in deze laatste periode gevormde veeren nog sterker het vrouwelijke type vertoonen, dan die welke gedurende de behandeling zijn ontstaan.

Dit blijkt uit het volgende:

1. In proef 1 krijgt hen N^0. 26 tijdens de tweede proefperiode, waarbij ze in plaats van Menformon slechts olie krijgt, eerst bruine veeren met slechts weinig zwarte vlekken, eerst langzamerhand worden de veeren steeds zwarter, zoodat bij het *einde* van de proef, zooals boven werd meegedeeld, zeer sterk zwart gevlekte bruine veeren aanwezig zijn.

Op dezelfde wijze gedraagt zich *na* de Menformon-behandeling hen 35 volgens waarnemingen in September gedaan.

De verschillende veranderingen ziet men beter, dan uit onze proefverslagen, uit bijgaande teekening (fig. 1) waarin 5 verschillende stadia bij een dier worden afgebeeld.

De twee hanen 17 en 22, die bij het einde van de tweede proefperiode op 8.IX. dus 4 weken na de laatste Menformon-inspuiting, op de in het begin van deze periode in het midden geplukte borst zwarte veeren hadden, waren aan het einde van de eerste proefperiode begonnen te ruien, aan den kant, die niet was geplukt. Zij kregen hier eerst bruin gevlekte veeren, dit was ook reeds het geval op het midden van de borst; toen evenwel na drie weken na staken van de Menformon-toediening opnieuw de borst geplukt werd, werden de nieuwe veeren volkomen zwart.

Bij de dieren van proef 2 werd na de Menformon-behandeling de rechter zijde gedeeltelijk door plukken, gedeeltelijk door ruien kaal. De daarna gevormde veeren zijn bij de behandelde hennen niet slechts bruin, maar zelfs helderder bruin dan de veeren van de andere zijde, waarvan reeds vóór de behandeling de veeren waren geplukt.

Ook bij de hanen is de feminiseering rechts sterker dan links, eerst drie weken later werden de op dit oogenblik geplukte veeren door de voor den castraat typische zwarte veeren vervangen.

Tenslotte was in proef 3 bij de niet gecastreerde hanen de nawerking van de behandeling het duidelijkst: hier waren n.l. die borstveeren der rechter zijde — die intusschen geruid had, — welke gedurende deze naperiode van ongeveer drie weken uitgegroeid waren, helder bruin, nog veel sterker dan de veeren van de geplukte zijde, die tijdens de Menformon-behandeling waren ontstaan. Zoo vonden wij op 1.X. nog jonge veeren van dit type. De dieren vertoonen op dit oogenblik een echt gynandromorphisme, daar zij een gedeeltelijk bruine hennenborst, geen zadelveeren en

Fig. 1. 5 verschillende stadia (1 – 5) van de gecastreerde met Menformon behandelde hen N⁰. 35. Elk volgend stadium is ontstaan, nadat de betreffende zone van te voren opnieuw is geplukt.
De op teekening V aangegeven data geven aan, hoe oud de veeren minstens zijn.

Fig. 2. Normale (ongecastreerde) haan N⁰. 68.
1. Bruine henneveeren ontstaan na staking der Menformon-behandeling en na spontane rui (nawerking).
2. Zwarte haneveeren, onstaan op die plekken, waar de rui pas later optrad.

een hanenkam bezitten. Zie Fig. 2. Wat de kam betreft, deze is tijdens de vijf weken durende behandeling in plaats van op normale wijze te groeien, eerder iets kleiner geworden (antimasculine werking). In de tweede helft der waarnemingsperiode trad een sterkere groei op.

Om practische redenen (mogelijkheid van grootere concentraties in olie dan in water) werden de tot dusverre beschreven proeven verricht met behulp van Menformon opgelost in olijfolie. Hoewel reeds uit de contrôle-proeven genoegzaam blijkt, dat de olie in dit opzicht als onschuldig is te beschouwen, leek het ons noodig het verschijnsel van kunstmatige henneveeren ook te voorschijn te roepen met oplossingen van *Menformon in water.*

Proef 4.

Op dit oogenblik is deze proef nog niet tot een eind gebracht. In behandeling zijn o.a. de hennen 31 en 43. De dieren stammen uit hetzelfde broedsel als die in de eerste beide proeven, zijn ter zelfder tijd gecastreerd en zijn reeds in proef 2 gebruikt.

De ingespoten preparaten zijn ook extracten uit urine van zwangere vrouwen, zij bevatten 1 M.E. in 0.003 resp. 0.001 mg en 40 M.E. resp. 50 M.E. per cc water.

Dosis van 3.X—10.X. 2 maal daags 3 cc à 40 M.E. = 2 × 120 M.E. van 11.X—17. X. 2 maal daags 1½ cc à 50 M.E. = 2 × 75 M.E. van 18. X of 2 maal daags 2 cc. à 50 M.E. = 2 × 100 ME.

Resultaat: Bij deze hennen zijn aan de geplukte linkerzijde helder bruine veeren te zien.

Proef 5.

Eveneens is nog in gang een proef met een menformon bevattend extract van mannenurine, dat reeds wat betreft het mannelijke hormoon (door den groei van den kam) werkzaam is gebleken.

Dit preparaat bevat 100 M.E. Menformon per cc olie (1 M.E. = 0.08 mg). De proef begon 3.X met twee hennen Nr. 32 en 42. Deze werden beide reeds in proef 2 gebruikt, de eerste twee als contrôles en de laatste behandeld met Menformon.

Zij stammen uit hetzelfde broedsel en werden gelijktijdig gecastreerd.

Dosis: 2 maal daags ½ cc à 50 M.E.

Resultaat: De veeren der hennen zijn op het oogenblik helder bruin, dus goed gefeminiseerd.

In deze mededeeling gaan wij niet in op de gewichtsverschillen en veranderingen der dieren gedurende de proeven, welke wij maandelijks hebben bepaald. De daaruit te maken gevolgtrekkingen, mogen hier echter nog kort worden vermeld.

1. De gewichten der hanen zijn ongeveer 20 % hooger dan die der even oude hennen.

2. De vol werkzame dosis voor de hennen 26 en 35 was relatief kleiner dan de onvolkomen werkzame dosis voor hanen, die kleiner waren dan deze hennen.

3. Een vergelijking tusschen de gewichten van behandelde en onbehandelde dieren doet een groote overeenstemming zien. Dit geldt zoowel

van gecastreerde als ongecastreerde dieren. Daaruit volgt de volledige ongiftigheid van biologisch werkzame doses van Menformon ook bij vogels, zooals die ons reeds bij zoogdieren bekend was.

Samenvatting.

Een invloed van Menformon is niet alleen op zoogdieren maar ook op vogels aan te toonen. Hier is in het bijzonder, behalve de slechts aangeroerde anti-masculinewerking, de invloed op het *veerenkleed* [1]) van hoenders nagegaan, waardoor ook de questie, of dit door het geslachtshormoon wordt beïnvloed, althans wat betreft het vrouwelijk hormoon is beslist.

De werking van Menformon uit zich bij het door ons gebruikte hoenderras (patrijs leghorn) in dien zin, dat bij verwijdering van het bestaande veerenkleed altijd een min of meer vrouwelijk, maar geen neutraal resp. mannelijk type veeren optreedt en wel niet alleen bij gecastreerde dieren van beiderlei kunne maar ook bij normale hanen.

(*Zusammenfassung.* Ein Einfluss von Menformon ist nicht allein bei Säugetieren sondern auch bei Vögeln aufzuweisen. In der vorliegenden Mitteilung ist abgesehen von der nur gelegentlich erwähnten antimaskulinen Wirkung besonders der *Einfluss auf das Federkleid* von Hühnern verfolgt. Hierdurch ist auch die Frage, ob dies von dem Sexualhormon beeinflusst wird, entschieden, jedesfalls hinsichtlich des weiblichen Hormons.

Diese Wirkung äussert sich bei der von uns benutzten Rasse (Leghorn doré) in dem Sinne, dass nach Entfernung des ursprünglich vorhandenen Federkleids ein mehr oder weniger weibliches auftritt, aber kein neutraler bezw. männlicher Federtypus. Dies gilt nicht allein für kastrierte Tiere beiderlei Geschlechts sondern auch für normale Hähne).

Pharmaco-therapeutisch Laboratorium der Universiteit.

Amsterdam, October 1929.

[1]) Op het Physiologencongres in Boston deelde Prof. ZAVADOWSKY uit Moskou op 23 Augustus mede, dat het hem niet gelukt was met zelf bereid „Menformon" een invloed op het veerenkleed te bereiken. In de discussie bij deze voordracht kon één van ons (E. L.) reeds over positieve resultaten, die intusschen in Amsterdam waren bereikt, mededeeling doen. Zoowel Prof. ZAVADOWSKY als wij zijn van oordeel, dat het bij deze negatieve resultaten waarschijnlijk gelegen heeft aan te geringe hoeveelheden Menformon, of misschien aan een niet juiste verdeeling van de dosis.

Physiology. — *System der optischen und taktilen Raumtäuschungen.* Von G. RÉVÉSZ. (Communicated by Prof. BROUWER.)

(Communicated at the meeting of October 26, 1929).

I.

Obwohl man die geometrisch-optischen Täuschungen Jahrzehnte lang untersucht und mit viel Eifer nach neuen Täuschungserscheinungen gesucht hat, lässt sich doch nicht bestreiten, dass die theoretische Aufklärung dieser bekannten Erscheinungen bis zum heutigen Tage nicht geglückt ist. Die auf einheitlichen Prinzipien ruhenden Theorien gehören zum Teil heute der Geschichte an, zum Teil schulden sie den Beweis, dass alle räumlichen Täuschungserscheinungen von den zugrunde gelegten Hauptgesichtspunkten aus wirklich zu erklären sind. Man kann ohne Uebertreibung behaupten, dass die riesige Mühe, die seit OPPEL, ZÖLLNER, POGGENDORF, HELMHOLTZ, HERING auf diesem Gebiete verwendet wurde, das Erfahrungsmaterial zwar in erheblichem Masse vermehrte und dem schon beinahe unübersehbaren Gebiet schärfere Grenzen zog, aber das Grundproblem in nennenswerter Weise nicht befördert hatte. Die Täuschungen lassen sich heute kaum leichter mit den übrigen optischen und raumpsychologischen Erfahrungen in Einklang bringen, wie damals ; sie werden noch vielfach als gelegentliche Besonderheiten unserer räumlich-optischen Wahrnehmung betrachtet, sie bleiben Gegebenheiten, die in das ziemlich geschlossene System unseres visuellen Raumbildes schwer hineinpassen.

II.

Ich glaube, es ist mir infolge einer kleinen Entdeckung gelungen den geometrisch-optischen Täuschungen den richtigen Platz in der Wahrnehmungslehre zuzuweisen und gleichzeitig die Richtung anzugeben, auf dem wir zu einer umfassenden und einheitlichen Theorie der Raumtäuschungen gelangen können. Vorwegnehmend kann ich sagen, dass dieser Weg über das neuerschlossene Gebiet der taktilen Raumtäuschungen führt.

Bei unserem Problem war der Ausgangspunkt : *die Beziehung der raumoptischen Eindrücke zu den raumtaktilen zu erforschen.*

Diese Frage lässt sich unter gewöhnlichen Bedingungen nicht beantworten, schon wegen der mannigfaltigen Verbindungen, die sich zwischen visuellen und taktilen Eindrücken geknüpft haben. Infolge dieser Beziehungen treten Tast- und Gesichtswelt einander immer näher, vertreten sich immer häufiger, verschmelzen sogar manchmal. Obgleich die phänomenale Verschiedenheit beider Sinnesgebiete deutlich zum Bewusstsein

gebracht werden kann, und die ineinandergreifenden optischen und taktilen Inhalte sich auch bis zu einem gewissen Grade voneinander scheiden lassen, dennoch entstehen grosse Schwierigkeiten, sobald man die Leistungen jeder der beiden Sinnen für sich getrennt prüfen will.

Die Aufgabe war also nach einer Erscheinungsgruppe umzusehen, die zwar beiden Sinnesgebieten gemeinsam angehört, und dennoch in jedem Sinnensgebiete selbständig auftritt, ohne Beeinflussung durch den anderen Sinn.

Auf dem ersten Blick scheint ein solches Gebiet innerhalb unserer Tastwahrnehmungen nicht vorzuliegen. Prüft man jedoch die raumpsychologischen Fragenkomplexe von dieser Fragenstellung aus, dann findet man, dass das Gebiet, welches unserer Voraussetzungen entspricht, das der *taktilen Raumtäuschungen* ist, — falls und soweit sie den optischen analog sind.

III.

Die erste Frage lautet also: Gibt es Täuschungen der Raumwahrnehmungen im taktilen Gebiet?

Die zweite Frage: Wenn es solche gibt, sind sie den optischen analog oder gar mit ihnen identisch?

Die drifte Frage wäre dann: Lassen sich im Taktilen alle Typen der geometrisch-optischen Täuschungen nachweisen oder beschränken sich die taktilen Raumtäuschungen nur auf einige dieser Typen oder gar auf einzelnen Täuschungsfiguren?

Die vierte Frage würde lauten: Sind die geometrisch-taktilen Täuschungen denselben Gesetzen unterworfen wie die geometrisch-optischen?

Als fünfte Frage ergäbe sich: Lässt sich die Unabhängigkeit der taktilen Täuschungen von den optischen in unanfechtbarer Weise nachweisen?

IV.

Unsere Forschungshypothese hat sich bei diesem Problem bewährt. Wir konnten alle unsere Fragen *positiv* beantworten.

Ich liess die bekannten Täuschungsmuster, welche in einer Grösse von 4—8 cm Länge und entsprechende Breite reliefartig auf einer Unterlage angefertigt waren, sowohl mit Hilfe von Bewegungen abtasten, wie auch durch simultanes Auflegen der Hand auf das Muster beurteilen.

Es liessen sich folgende Tatsachen feststellen:

Erstens: Es gibt ebenso wie im Optischen auch im Taktilen Raumtäuschungen, d.h. geometrisch-taktile Täuschungen, die ebenso leicht nachweisbar und ebenso prägnant in Erscheinung treten, wie die optischen [1]).

[1]) Bereits früher wurden zwei taktile Täuschungen nachgewiesen, die den geometrisch-optischen Täuschungen analog waren, jedoch ohne die theoretische Bedeutung dieser Uebereinstimmung erkannt zu haben.

Zweitens: Den optischen Erscheinungen gehen vollkommen entsprechende taktile parallel.

Da nun festgestellt ist, dass die räumlichen Täuschungen der beiden Sinnesgebieten ihrem Wesen nach identisch sind, ist es nicht mehr zulässig die geometrisch-optischen Täuschungen als ein besonderes Gebiet der visuellen Raumwahrnehmungen zu betrachten, sondern wir haben von nun an von *Raumtäuschungen überhaupt* zu reden, die sowohl durch den Gesichtssinn wie auch durch den Tastsinn in gleicher Weise vermittelt werden. Das steht nicht im Widerspruch mit der Tatsache, dass es Raumtäuschungen gibt, welche nur im Visuellen, während andere nur im Taktilen zu beobachten sind.

Drittens: Mit Ausnahme der perspektivischen Täuschungen und derjenigen, die durch den Bau und die besondere Funktion des Sehapparates bedingt sind (Verschiedenheit der Netzhautstelle, Accomodation, Irradiation, Augenbewegung, usf.), sind alle sonstigen Typen, die aus der Raumoptik bekannt sind, taktil nachweisbar. Es sind also sowohl die linearen- und Flächen-Grössentäuschungen, wie auch die Winkel- und Richtungstäuschungen, ferner die sog. Kontrasttäuschungen im Taktilen vertreten

Im Ganzen habe ich bisher ca. 30 geometrisch-optische Täuschungsfiguren verschiedener Typen im Taktilen untersucht. Um einige konkrete Beispiele zu nennen, führe ich folgende an: Kundtsche Täuschung, die bekanntlich die Ueberschätzung der ausgefüllten Linie gegenüber der nicht ausgefüllten demonstriert; Grössentäuschung durch wechselnde Begrenzung von Linien und Flächen; die bekannte Müller-Lyersche Streckentäuschung; die von Zöllner beobachtete Täuschung, wo parallele Geraden wegen kreuzender Striche abwechselnd gegeneinander schief erscheinen; Poggendorffsche Figur, welche die scheinbare Verschiebung der Fortsetzung eines Geraden infolge scheinbarer Winkelvergrösserung hervortreten lässt; Ueberschätzung vertikaler linearer- und Punktdistanzen; Trapez- und Segmenttäuschungen; scheinbare Krümmung einer geraden Tangente wegen Angleichung an eine objektive Krumme, schliesslich Fälle von Mengentäuschungen [1]).

Viertens: Die von uns untersuchten taktilen Täuschungen werden von den gleichen Tendenzen und Gesetzen beherrscht wie die optischen.

Fünftens: Es liess sich endgültig erweisen, dass räumliche Eindrücke sowohl optisch wie auch taktil fundiert sein können und dass sie in diesem Sinne gegenseitig unabhängig sind. Alle taktilen Täuschungen nämlich, die wir an uns normalen Vollsinnigen ermittelt haben, sind einerseits an Personen mit stark reduzierter und sogar aufgehobener visueller Vorstellungsfähigkeit, anderseits aber auch an Blindgeborenen mit voller Sicherheit nachzuweisen. Dass bei all diesen Personen, insbesonders bei Blindgeborenen Gesichtsvorstellungen und visuelle Erfahrungen die taktilen Eindrücke nicht beeinflusst haben können und infolgedessen für die räumliche Auffas-

[1]) Ausführliche Mitteilungen über diese Versuche nebst Behandlung zusammenhängender theoretischen Probleme erscheinen demnächst.

sung und Einordnung der Tasteindrücke in das Raumsystem nur ursprüngliche räumliche Qualitäten des Tastsinnes in betracht kommen können, liegt auf der Hand.

V.

Unsere Feststellungen führen zunächst zu folgenden vier Hauptsätzen der Lehre der Raumwahrnehmungen:

1. Sowohl der Gesichtssinn wie auch der Tastsinn vermögen Raumanschauung und konkrete Raumeindrücke und Vorstellungen unabhängig und selbständig voneinander zu vermitteln.

2. Geometrische Konfigurationen werden vom Gesichtssinn und vom Tastsinn in gleicher Weise erfasst und bei der Gestaltauffassung prinzipiell in gleicher Weise verwertet.

3. Raumformen bildende und vor allem Raumformen und Raumelemente modifizierende Tendenzen stimmen in beiden Gebieten überein.

4. Prinzipiell gibt es nur *eine* Raumanschauung. Gesichtsraum und Tastraum sind phänomenale Auswirkungen derselben.

Die Sätze 2 und 3 beziehen sich, — entsprechend des Geltungsbereichs der geometrisch-optischen Täuschungen, — zunächst auf zweidimensionale Gebilde. Weitere Untersuchungen sollen uns zeigen, wie weit sich diese und noch weitere Sätze auch für dreidimensionale Körperformen gelten.

Es hat sich also herausgestellt, dass die optischen und taktilen Eindrücke, — trotz ihrer phänomenalen Verschiedenheit, — in bezug auf das Erleben räumlicher Beziehungen bei zweidimensionalen Gebilden einander gleichen. Das optische Erlebnis in seinem phänomenalen Charakter kann nämlich von einem, auf demselben Gegenstand bezogenen taktilen Erlebnis recht verschieden sein, aber der Eindruck, der durch die Wahrnehmung der objektiven Raumverhältnisse entsteht, ist im Wesen gleich. Wir haben also anzunehmen, dass die formale Struktur der Raumelemente und bis zu einem gewissen Grade auch die der Gegenstände, die doch in hohem Masse von den geometrischen Verhältnissen abhängt, im Optischen und im Taktilen weitgehend übereinstimmt. Hieraus folgt mit Notwendigkeit, dass die Raumvorstellungen der Blinden, — innerhalb der Grenzen des greifbaren und tastbaren Nahraumes, — von den unsrigen nicht verschieden sein können. Denn wenn die Raumtäuschungen bei Blinden die gleiche sind wie bei uns Vollsinnigen, können die ihnen zu grunde liegenden räumlichen Eindrücke nicht verschieden sein.

Aus den dargelegten Sätzen folgt, dass nur eine solche Theorie den mannigfaltigen gemeinsamen Täuschungserscheinungen gerecht werden kann, welche die Raumtäuschungen aus den allgemeinen Funktionen der Raumwahrnehmung abzuleiten vermag. Der überwiegende Teil der räumlich- optischen und räumlich- taktilen Täuschungen geniessen also keine Sonderstellung innerhalb unserer Raumwahrnehmungen; sie stellen bloss besonders prägnante Aeusserungen jener Funktionen oder Tendenzen dar,

die auch bei der räumlichen Gestaltung unserer visuellen und taktilen Eindrücke entscheidend sind. Für Festlegung der allgemeinen Raumwahrnehmungsfunktionen bieten gerade die Raumtäuschungen die wichtigsten theoretischen und experimentellen Gesichtspunkte.

Es sei noch erwähnt, dass durch unsere Ergebnisse die alte Streitfrage um den Nativismus und Empirismus in einem neuen Licht erscheint.

Der Empirismus würde sich bei der Interpretation unserer Feststellungen in eine Gewebe sonderbarer Hypothesen verlieren. Kaum jemanden würde es gelingen, die Uebereinstimmung der taktilen Raumtäuschungen mit den optischen von dieser Lehre aus begreiflich zu machen. Weder die blosse Existenz der taktil-räumlichen Täuschungen, noch weniger ihre Uebereinstimmung mit den entsprechenden optischen Illusionen lassen sich auf Erfahrung, auf assoziativen Vorgänge zurückführen.

Wenn wir aber an der angeborenen Natur der Raumanschauung, an ihrer Ursprünglichkeit festhalten, damit scheint nur eine Erfahrungsmöglichkeit vereinbar zu sein, nämlich, dass der Gesichtssinn wie der Tastsinn in gleicher Weise der einheitlichen Raumanschauung teilhaftig sind, woraus folgt, dass die Raumverhältnisse in beiden Sinnesgebieten, — allerdings mit der Einschränkung, die der Funktionsverschiedenheit der beiden Sinnesorganen begründet ist, — in analoger Weise in Erscheinung treten müssen. wie es uns bei den Raumtäuschungen nachzuweisen in der Tat gelungen ist.

Botany. — *Growth and inheritance of leafdimensions in the broad-bean.* (Vicia Faba L.) By M. J. SIRKS. (Communicated by Prof. J. C. SCHOUTE.)

(Communicated at the meeting of October 26, 1929).

The ideas of GOLDSCHMIDT concerning the quantitative nature of genes have drawn the geneticists attention again to the desirability of deepening our knowledge of the inheritance of so-called quantitative characters. Since the initial important publications by NILSSON-EHLE (1909—1911) and by EMERSON and EAST (a.o. 1913) quantitative inheritance has been rather neglected, as contrasted with qualitative heredity. This is mainly caused by the much more comprehensive work, required by an exact study of quantitative characters. For qualitative characters in most cases may be judged directly at sight, while quantitative differences by their greater and often transgressive modificability require a much more complicate technique of measurement or weighing.

It is rather questionable how far the hypothesis of multiple factors, at present generally assumed to explain the inheritance of quantitative characters indeed is the right one and this ought to be considered in future with greater severity; as I have pointed out in 1926 the absolutism of this hypothesis is rather subject to doubt. And the data in this paper seem to prove that multiple factors as well can play a prominent part in the inheritance of quantitative characters.

An attempt to analyse the various races of the broad bean (Vicia Faba L.) from the genetical point of view has been undertaken in 1916 and the researches referring to this have now been closed for the main part after a period of thirteen years. The results are in preparation; I hope they will be published as soon as possible in a more or less monographic form. Though the species mentioned seemingly produces rather unfavourable materials for a genotypic analysis, it has the more value for the problems of quantitative inheritance as most characters which distinguish the various races, are of a quantitative nature.

The length of plants, number of stalks, form, dimensions and even colour of leaves, flowers, fruits and seeds, they all belong for the greater part to the group of quantitative characters and therefore they produce useful data in this sense.

In this paper a preliminary communication will be made on the shape and the dimensions of the leaflets, which make up the compound leaves of this species; the results of it seem to lead to rather important conclu-

sions. Strains of the following eight races have been used especially for the studies of these characters:

1. Groote groene Engelsche Windsor = Large green Windsor.
2. Groote blanke Engelsche Windsor = Large white Windsor.
4. Langschee witbloeiend witblijvend = Long pod, white flowered, white seeded.
5. Driemaalwit = Triple white.
6. Echte vroege breede Haarlemmer = Real early broad Haarlem.
8. Echte vroege breede witkiem = Real early broad white nose.
11. Mazagan vroege kleine inmaak = Mazagan early small canning.
14. Fairbards dwarf prolific.

In the following these races will be mentioned by their number only, to evade the unpractical, long names. Moreover one should keep in mind, that this number does not represent the race as a whole, but only the strain, bred from it for the purpose of genetical research; all commercial races of this species are always rather impure while they contain a number of strains with different qualities, and besides sometimes the same name is used in commerce for entirely different populations. Thus the varietal name as such is of minor importance. In this paper the principal data and conclusions only will be given; complete documentation is reserved for the monographic compilation.

The strains show remarkable differences between them in shape and dimensions of the leaflets; beside very regular elliptical leaflets of various dimensions, typical ovate and rather scarce obovate types were observed. These types of leaflets as such could not be considered as an individual character, since on the same plant ovate, as well as elliptical and obovate leaflets may occur. Shape and dimensions are highly variable not only in the leaflets of the same leaf, but also according to the rank of this leaf. An exact study of the individual fluctuations among the leaflets showed thus to be the first necessary thing.

For that purpose thirty normal plants of each strain were used; all the leaflets of the leaves 2 up to 8 were measured for their width (W), and for both parts of their length: from the insert of the petiole to the line of greatest width (basal part, B) and from this line to the apex of the leaflet (terminal part, T). Of course these measurements were taken with the greatest possible accuracy; the dimensions were determined to the nearest 0.5 mm. and all the leaflets, which had been damaged by insects or some interruptions of growth, were eliminated. These abnormal leaflets did not count for more than 10 per cent of the total number. Some difficulty was met in determining the line of greatest width, as the leaflets were not always exactly symmetrical; for this the leaflets were placed between two parallel rulers, and the intersection of the main vein with the line connecting both points of contact with the rulers, was assumed to be the centre of the line of greatest width. From the data obtained the mean values were calculated unto 1/10 mm.; the numbers given in the tables relate to this unity. Moreover the proportions 100 T : B have

been calculated as an index for leafshape (100 T : B > 100 ovate; 100 T : B = 100 elliptical and 100 T : B < 100 obovate).

As regards the intrafoliar variability the data have been summarized in table 1 (fig. 1); four strains (6, 5, 4 and 8) had the maximum of

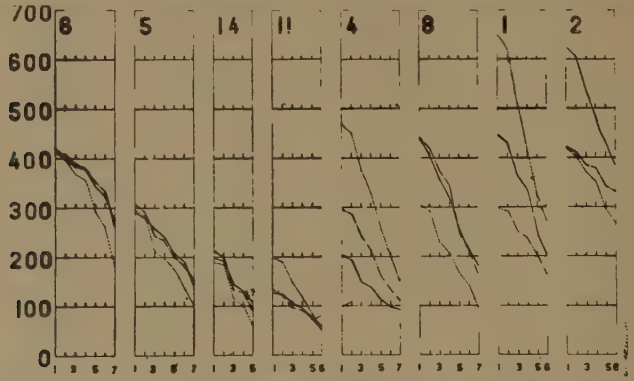

Fig. 1. Mean values of the width of leaflets (W) and of the basal (B) and terminal (T) parts of the length of all leaflets in leaves of the 6th. rank on thirty normal plants.

——————— Width.
——— — ——— — Basal part of length.
— — — —¼— — — Terminal part of length.

seven leaflets in each leaf; three (11, 1 and 2) six only, while the last one (14) never produced more than 5 leaflets. The mean values, given in this table, lead to some preliminary conclusions: from basis towards the apex the leafshape is changed in this sense, that the 100 T : B-proportion decreases continuously. In strain 1 this proportion goes down from 218 to 165; in 4 from 159 to 134; in 11 from 148 to 87; in 6 from 101 to 70; in 2 from 100 to 80; in 5 from 98 to 67; in 14 from 92 to 66 and in 8 from 69 to 59. It looks like if the basal leaflets only are fullgrown, while the other ones have not yet reached their maximum of development. The eight strains seemed to belong to four different types, as far as this T: B-proportion is concerned; strain 1 is ovate with 218 and strains 4 and 11 are ovate with about 150; strains 6, 5, 14 and 2 seem to be elliptical with about 100, while strain 8 is obovate with 69. In the beginning much attention has been drawn to these T : B.-proportions, because of the seemingly geometrical progression ar^3, ar^2, ar and a, in which a was about 66.66 and x = 1.5, but later studies showed that these proportions can be the result of entirely different genotypic constitutions. Moreover the falling off in strain 4 was quite different from that in strain 11, though in both the basal leaflets had a 100 T: B.-proportion of about 150. Anyhow these data lead to the conclusion, that the dimensions of the basal leaflets may serve as a guidance for further measurements.

A comparison of the dimensions of the basal leaflets in the leaves 2

TABLE 1. Mean values of the width of leaflets (W) and of the basal (B) and terminal (T) parts of the length of all leaflets in leaves of the 6th rank on thirty normal plants. D = difference.

Strain	Dim.	Basis 1	2	3	4	5	6	Apex 7
6	W	418	403	386	380	343	324	274
	D		15	17	6	37	17	50
	B	412	407	389	382	351	332	262
	D		5	22	7	31	19	70
	T	415	399	368	356	292	263	183
	D		16	31	12	64	29	80
	100 T : B	101	98	95	93	83	79	70
5	W	292	278	257	241	206	191	142
	D		14	21	16	35	15	49
	B	304	290	246	233	201	179	152
	D		14	44	13	32	22	27
	T	298	293	213	198	174	137	103
	D		5	80	15	24	37	34
	100 T : B	98	101	86	85	86	76	67
14	W	199	193	142	131	107	—	—
	D		6	51	11	24		
	B	209	198	135	129	94	—	—
	D		11	63	6	35		
	T	192	184	116	101	62	—	—
	D		8	68	15	39		
	100 T : B	92	93	86	78	66	—	—
11	W	126	121	103	96	72	53	—
	D		5	18	7	24	19	
	B	133	120	97	93	77	56	—
	D		13	23	4	16	21	
	T	198	190	148	116	83	49	—
	D		8	42	32	33	34	
	100 T : B	148	158	153	124	108	87	—
4	W	201	193	147	138	113	98	94
	D		8	46	9	25	15	4
	B	294	286	243	214	161	134	111
	D		8	43	29	53	27	23
	T	467	451	376	327	253	199	149
	D		16	75	49	74	54	50
	100 T : B	159	158	154	153	157	148	134
8	W	435	408	356	318	256	217	182
	D		27	52	38	62	39	35
	B	439	419	372	344	249	212	163
	D		20	47	28	95	37	49
	T	305	291	229	206	167	141	97
	D		14	62	23	39	2	44
	100 T : B	69	69	61	60	67	66	59
1	W	445	431	362	315	242	203	—
	D		14	69	47	73	39	
	B	296	293	250	237	202	165	—
	D		3	43	13	35	37	
	T	647	622	498	415	320	273	—
	D		25	124	83	95	47	
	100 T : B	218	209	199	176	158	165	—

TABLE I (*continued*).

Strain	Dim.	Basis 1	2	3	4	5	6	Apex 7
2	W	624	607	532	484	411	382	—
	D		17	75	48	73	29	
	B	419	410	384	378	343	331	-
	D		9	26	6	35	12	
	T	418	401	370	351	296	267	—
	D		17	31	19	55	29	
	100 T : B	100	98	96	92	83	80	—

up to 8 of the same plants (table 2 and fig. 2) shows that the basal leaflets in the successively developed leaves 2, 3 and 4 seem to be interrupted in their process of growth: they donot reach a fullgrown

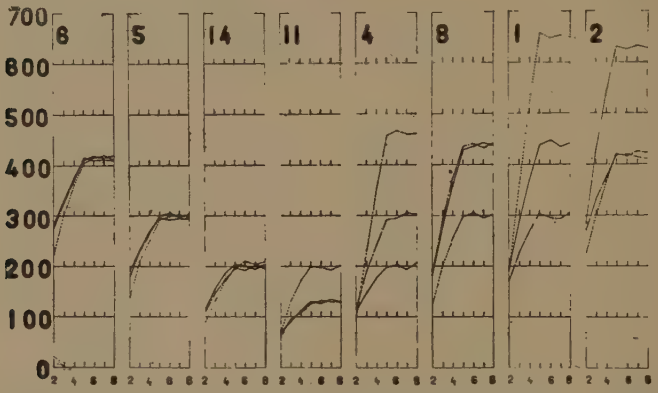

Fig. 2. – Mean values of the width of leaflets and of the basal and terminal parts of the length of the basal leaflets in well-developed leaves on thirty normal plants.

——————— Width.
——— — ——— — Basal part of length.
— — — — — — — Terminal part of length.

state. The T: B-proportions and the absolute measurements show a continuous increase in these leaves, while in the fifth leaf they have gained a definite value, which is retained until the leaves of the eighth rank inclusive. This apparent stability in leafshape in the leaves 5 to 8 has led to the conclusion that an individual may be characterised most strictly by the mean values of measurements of the basal leaflets in these leaves. The T : B-proportions, which showed an increase in the older leaves, remain rather fixed from the fifth leaf on.

In order to get a view on the factors, which cause the building up of these seemingly complicated phenomena, researches were made in two directions :

1. Fifteen plants of each strain were measured during their development for the three dimensions in the basal leaflets of the 5th., 6th., 7th. and 8th.

TABLE 2. Mean values of the width of leaflets (W) and of the basal (B) and terminal (T) parts of the basal leaflets in well-developed leaves 2—8 on thirty normal plants. D = difference.

Strain	Dim.	Basis 2	3	4	5	6	7	Apex 8
6	W	281	333	378	411	418	416	419
	D		52	45	33			
	B	275	329	374	413	412	415	414
	D		54	45	39			
	T	224	304	359	407	415	419	413
	D		80	55	48			
	100 T : B	81	92	96	98	101	101	100
5	W	191	234	270	297	292	294	293
	D		43	36	27			
	B	180	231	269	299	304	296	301
	D		51	38	30			
	T	139	209	254	293	298	302	295
	D		70	45	39			
	100 T : B	77	90	94	98	98	102	98
14	W	117	155	186	204	199	194	205
	D		38	31	18			
	B	111	146	175	196	209	203	197
	D		35	29	21			
	T	88	132	170	197	192	201	210
	D		44	38	27			
	100 T : B	80	90	97	100	92	99	105
11	W	71	93	114	130	126	134	129
	D		22	21	16			
	B	66	95	109	124	133	127	128
	D		29	14	15			
	T	72	133	174	201	198	195	202
	D		61	41	27			
	100 T : B	110	140	160	162	150	154	158
4	W	113	149	178	199	201	195	206
	D		36	29	21			
	B	117	192	246	293	294	304	301
	D		75	54	47			
	T	99	232	356	458	467	460	463
	D		133	124	102			
	100 T : B	85	121	137	156	159	151	153
8	W	182	277	360	431	435	442	437
	D		95	83	71			
	B	186	288	373	436	439	433	444
	D		102	85	63			
	T	114	200	255	299	305	297	302
	D		86	55	44			
	100 T : B	61	70	68	68	69	68	68
1	W	195	292	368	439	445	433	441
	D		97	76	71			
	B	171	229	270	302	296	295	306
	D		58	41	32			
	T	190	368	531	656	647	653	649
	D		178	163	135			
	100 T : B	111	160	197	217	219	221	212

TABLE 2 (*continued*).

Strain	Dim.	Basis 2	3	4	5	6	7	Apex 8
2	W	283	425	544	629	624	631	627
	D		142	119	85			
	B	269	332	379	420	419	413	422
	D		63	47	41			
	T	225	301	367	421	418	426	423
	D		76	66	54			
	100 T : B	83	90	97	100	100	103	100

leaves ; the growth appeared to be completed in about 15 days. The mean values for these plants have been calculated and summed up in table 3 (figs. 3 and 4).

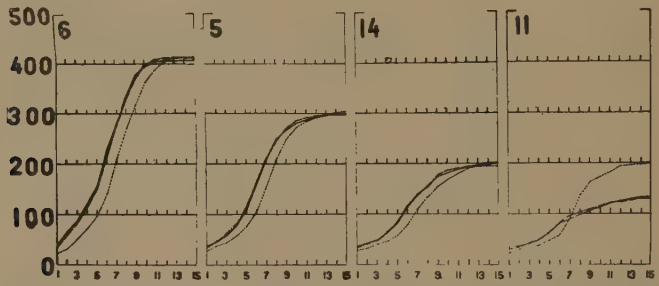

Fig. 3. Mean values of the width and of the basal and terminal parts of the length of basal leaflets in the fifth to eighth leaves on fifteen plants, measured during their development. Strains 6, 5, 14 and 11.

———————— Width.
———— — ———— — — Basal part of length.
— — — — — — — — Terminal part of length.

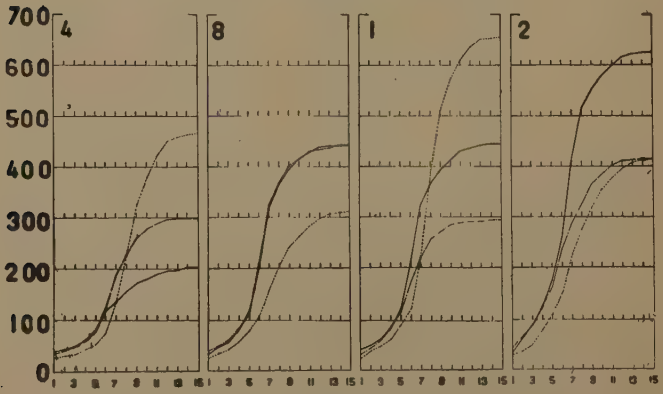

Fig. 4. The same as in fig. 3 for strains 4, 8, 1 and 2.

———————— Width.
———— — ———— — — Basal part of length.
— — — — — — — — Terminal part of length.

TABLE 3. Mean values of the width of leaflets (W) and of the basal (B) and terminal (T) parts of the length of basal leaflets in the 5th to 8th leaves on 15 plants, measured during their development. I = daily increase.

Strain	Dim.	Day 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
6	W	38	62	84	107	156	223	279	331	375	393	405	406	406	407	407
	I		24	22	23	49	67	56	52	44	18	12	1	0	1	0
	B	36	59	83	109	153	221	280	329	370	394	408	411	411	411	411
	I		23	24	26	44	68	59	49	41	24	14	3	0	0	0
	T	23	31	51	72	97	142	213	275	319	363	385	404	412	413	413
	I		8	20	21	25	45	71	62	44	44	22	19	8	1	0
	100 T : B	64	53	61	66	63	64	76	83	86	92	94	98	100	100	100
5	W	35	48	59	79	109	161	204	248	265	280	289	295	297	297	297
	I		13	11	20	30	52	43	44	17	15	9	6	2	0	0
	B	37	47	64	82	111	163	208	246	270	284	292	296	299	299	300
	I		10	17	18	29	52	45	38	24	14	8	4	3	0	1
	T	29	38	44	59	77	104	157	206	249	273	284	294	299	301	304
	I		9	6	15	18	27	53	49	43	24	11	10	5	2	3
	100 T : B	78	81	68	72	70	64	76	84	92	96	97	99	100	101	101
14	W	36	42	49	62	81	113	137	157	174	181	189	195	198	198	200
	I		6	7	13	19	32	24	20	17	7	8	6	3	0	2
	B	34	41	47	63	83	116	138	156	171	185	190	196	196	197	197
	I		7	6	16	20	33	22	18	15	14	5	6	0	1	0
	T	28	32	40	46	58	77	109	134	155	168	181	192	196	199	201
	I		4	8	6	12	19	32	25	21	13	13	11	4	3	2
	100 T : B	82	78	85	73	70	67	79	86	90	91	95	98	100	101	102
11	W	33	36	40	48	61	80	90	98	107	113	120	125	129	131	131
	I		3	4	8	13	19	10	8	9	6	7	5	4	2	0
	B	31	35	41	50	62	82	96	102	109	116	122	124	127	128	128
	I		4	6	9	12	20	14	6	7	7	6	2	3	1	0
	T	23	29	33	38	47	60	92	137	162	173	182	194	196	198	199
	I		6	4	5	9	13	32	45	25	11	9	12	2	2	1
	100 T : B	74	83	80	76	76	73	96	134	149	149	149	156	154	155	156

TABLE 3 (*continued*).

Strain	Dim.	Day 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	W	38	44	48	63	83	117	136	156	172	180	190	196	198	202	202
	I		6	4	15	20	**34**	19	20	16	8	10	6	2	4	0
	B	35	41	49	62	80	119	187	228	261	279	288	296	299	299	299
	I		6	8	13	18	39	**68**	41	33	18	9	8	3	0	0
	T	26	30	33	43	52	73	137	243	326	379	421	449	458	463	465
	I		4	3	10	9	21	64	**106**	83	53	42	28	9	5	2
	100 T : B	74	73	67	69	65	61	73	107	125	135	146	152	153	155	155
8	W	34	46	57	80	119	202	319	362	395	418	429	437	439	439	439
	I		12	11	23	39	83	**117**	43	33	23	11	8	2	0	0
	B	39	47	60	84	117	210	322	368	402	417	430	435	438	440	440
	I		8	13	24	33	93	**112**	46	34	15	13	5	3	2	0
	T	25	33	42	57	77	106	161	203	243	262	282	290	296	298	301
	I		8	9	15	20	29	**55**	42	40	19	20	8	6	2	3
	100 T : B	64	70	70	68	66	50	50	55	60	63	66	67	67	67	68
1	W	41	48	61	82	121	206	322	368	392	411	430	438	442	444	444
	I		7	13	21	39	85	**116**	46	24	19	19	8	4	2	0
	B	32	42	58	84	118	174	221	252	270	283	289	291	291	292	292
	I		10	16	26	34	56	47	31	18	13	6	2	0	1	0
	T	23	35	46	57	87	124	222	386	509	573	613	636	650	652	655
	I		12	11	11	30	37	98	**164**	123	64	40	23	14	2	3
	100 T : B	72	83	79	68	73	71	100	153	188	203	212	219	223	223	224
2	W	34	60	89	123	174	263	409	512	552	579	600	615	623	624	624
	I		26	29	34	51	89	**146**	103	40	27	21	15	8	1	0
	B	42	63	87	124	167	236	286	324	362	383	401	410	413	416	416
	I		21	24	37	43	**69**	50	38	38	19	18	9	3	3	0
	W	29	35	47	75	108	154	224	273	316	350	376	395	407	414	415
	I		6	12	28	33	46	70	49	43	34	26	19	**12**	7	1
	100 T : B	69	55	54	60	64	65	78	84	87	91	94	96	98	99	100

2. In the crosses and their offspring the definitive dimensions of these four basal leaflets were measured and a study was made of their segregations in the F_2-families and some F_3-families (cf. table 4).

TABLE 4. Probable groupfrequencies in the segregating families. G = growth factor. A = additional factor.

Cross	Parents	Dim.	G4A2 925	G3A2 660	G4A1 630	G2A2 465	G3A1 440	G4 420	G3 G2A1 G1A2 300	G2 G1A1 200	G1 130
1	1×14	W					78		45	6	
		B							100	29	
		T		69		21	5		28	6	
2	2×4	W			96		4	30	34	9	
		B			92		2	34	33	12	
		T	93	3	1	30	2	32	4	8	
5	4×6	W						131	2	45	
		B			96		1	35	33	13	
		T	94	1	4	35		33	2	9	
5a	6×4	W						86	2	33	
		B			66		1	20	27	7	
		T	62	1	3	25		21	3	6	
7	4×11	W								117	37
		B							85	59	10
		T				86			57	11	
7a	11×4	W								125	40
		B							95	62	8
		T				92			61	12	
8	4×14	W								172	
		B							128	44	
		T				125			3	44	
12	5×14	W							121	41	
		B							121	41	
		T							121	41	
13	2×1	W			118		38				
		B						118	38		
		T	91	27	2		3	25	8		
15	2×11	W			102		1	32	4	35	10
		B						134	2	3	45
		T			100		2	34	1	35	12
16	2×14	W			76		2	27	22	10	
		B						103	3	31	
		T						103	3	31	
19	1×6	W			83		26	27	10		
		B						110	36		
		T	80	26	6			24	10		
20	1×4	W					91		59	8	
		B					94		53	11	
		T		122		36					

TABLE 4 (*continued*).

Cross	Parents	Dim.	G4A2 925	G3A2 660	G4A1 630	G2A2 465	G3A1 440	G4 420	G3 G2A1 G1A2 300	G2 G1A1 200	G1 130
21	1×11	W					70		27	26	9
		B							95	4	33
		T		68		4	27		26	7	
22	6×11	W						99	2	1	33
		B						99	2	1	33
		T			72		1	27	1	24	10
23	6×14	W						85	5	29	
		B						85	5	29	
		T						85	5	29	
28	11×14	W								123	38
		B								123	38
		T							92	59	10
34	5×11	W							100	1	39
		B							100	1	39
		T					73		28	29	10
35	5×6	W						104	37		
		B						104	37		
		T						104	37		
36	8×11	W					60		23	23	8
		B					58		26	24	6
		T					62		20	25	7
37	8×2	W			94		31		7		
		B			69		24	25			
		T						94	31		
38	6×2	W			101			28			
		B						129			
		T						129			
39	1×5	W					113		40		
		B							153		
		T		110			5		38		
40	8×1	W					138				
		B					106		32		
		T		104			3		31		
41	5×8	W					84		25		
		B					83		126		
		T							09		

It ought to be premised that the want of adequate technical equipment made it impossible to consider the results of the growth-studies as physiologically entirely exact. The measurements were made for some individuals in a greenhouse, the minimum-temperature of which at night was about 10° C., while in the day-time a maximumtemperature of 28° C. was reached. For the greater part the measurements were taken in the open

field, where no temperatures could be recorded, but where the conditions owing to the continuously bright summer were favourable. After all other workers also have obtained rather sufficient results with various materials under not always uniform conditions (EMERSON, 1916; REED, 1919-1924). We may even cite the words of REED (1924, p. 348): "A plant growing in the field is in dynamic equilibrium with its surroundings, otherwise it is dead. If cloudy days or drought or other changes ensue, the activities of the plant take such a course, that the effect of the changed conditions is minimized, and the growth rate generally suffers little change." The growth-curves of the greenhouseplants when compared with those of the individuals belonging to the same strain in the open field did not show any important difference. As the data in table 3 and figs. 3 and 4 show, the surrounding conditions indeed have not caused any remarkable disturbance, but it is self-evident that these growth-curves donot answer the requirements of a physiologically exact study. Their conclusive force is limited to conclusions of a comparison between the strains used; they are unable to decide the question if they may be identical with the curves for autokatalytical processes (ROBERTSON, 1923) or if they are not (v. d. SANDE BAKHUYZEN, 1926, 1927).

The numbers given for each type in table 4 should be considered as rather approximative; in consequence of the modificability it is difficult to determinate the right dividing line between the various types, especially in the regions between 400 and 480 and that between 620 and 680. The modes of the frequencycurves however may always be chosen as a type and the adjacent groups have to be added in a more or less arbitrary way.

In the growth-curves for strains 6, 5 and 14 a striking regularity was observed; those for W (width) and for B (basal part) are almost identical; they reach their maximum of growth between the fifth and the sixth day of observation, and the finish on the twelfth day. The curves for T (terminal part) rise somewhat later, so that the maximal increase is observed between the sixth and the seventh day and the growth is completed on the 13th day. The T : B-proportions remain constant during six days and begin to change from the seventh day on. The three dimensions seem to be produced by the same cause, though this cause in the terminal parts works one day later than it does in W and in B.

However these strains show remarkable differences between them when compared for the steepness of the growthcurve and this gives the impression of quantitative differences in the factors, which are cause of the growthrate in the three leafdimensions. This impression is affirmed by the results of the crosses. In the F1-generations the growthfactor of greatest intensity seems to dominate rather completely; in the F2-generations the dimensions for W, B and T in the same individual are always the same and the segregations suit entirely to the monohybrid 3 : 1-scheme. The genotypic factors which underly these growthcurves may be called G4, G3, and G2, and the F2-generations segregated in cross 35 (5×6) into 104 G4 :

37 G3; in cross 12 (5 × 14) into 121 G3 : 41 G2 and in cross 23 (6 × 14) into 85 G4 : 5 G3 : 29 G2.

The conclusion may be drawn, that here a series of three multiple allelomorphs are at work with almost complete dominance of the most intensive factor; that these triple allelomorphs are different in quantitative nature and that the two extremes of this series (G4 and G2) can produce a few exceptional intermediate individuals (G3). To me it seems to be impossible to give an other interpretation of these facts; they cannot be explained with the aid of multiple factors.

When strain 11 is now drawn into our attention, the growthcurves of this strain for W and B appear to follow the same line as those in the strains above-mentioned, but their steepness is still slower (G1-type). The growthcurve for T however deviates from this type. During six days it is parallel to those of W and B, though again one day later; on the seventh day the curve begins to show a sudden remarkable rise so that its regular course is broken. At first sight one could assume that the curve changes at once from the G1-type into the G2; the ultimate result is the same T-value as shown by strain 14. The cross with this type (cross 28, 11 × 14) produced for W and for B a monohybrid segregation (123 : 38), in consequence of which the G1-factor may be added to the above series of multiple allelomorphs. For T however the F1-generation exceeded both parental strains and in F2 a dihybrid segregation has been observed (92 : 59 : 10 or about 9 : 6 : 1). The frequency-curve for F2 showed three distinct modes: one in G1, one in G2 (which cannot be distinguished phenotypically from the 11-type) and one in the class, in which the F1-individuals had been found.

It seems to me obvious, that in strain 11 a common growthfactor G1 is responsible for the growthrate in W, B and T and that besides in T an additional factor may be assumed, whose activity begins about the seventh day of observation and adds a considerable part to the length of the terminal region. This additional factor may be called T1. The formula for strain 14 would be G2t and that for strain 11 G1T1, which gives rise to a F1-generation G2T1 and a dihybrid segregation (G2T1, G2t, G1T1 and G1t). Because of the phenotypical identitiy of G2t and G1T1 in F2 a frequencycurve results with three modes and a segregation 9 : 6 : 1.

This assumption receives full corroboration from the other crosses in which strain 11 has been used, viz. crosses 22 (6 × 11) and 34 (5 × 11). In both crosses the F2-generations for W and B show a bimodal curve with their modes in both parental classes, though in both again some exceptional intermediate types appeared. For these dimensions a monohybrid segregation is most plausible. On the other hand both crosses produced F1-generations with highly increased T-values when compared with the parents. The F2-generation from cross 34 gave a trimodal curve which perhaps may be grouped into four classes (73 : 28 : 29 : 10). Cross

22 (6 × 11) showed four distinct modes with a few intermediate individuals (72 : 1 : 27 : 1 : 24 : 10). This greater clearness in segregation was caused by the more diverging values of the parental plants, wherefrom in the F2 the resulting types G4T1, G4t, G1T1 and G1t could be distinguished with greater certainty.

In strain 4 the three growthcurves are all different. The curve for W seems to be identical with the G2-curve from strain 14; that for B resembles the G3-curve in strain 5, while the T-curve is still steeper than the G4-curve which was observed in strain 6. Here again the three curves run parallel with the G2-curves until the seventh day of observation; the rise of the B- and T-curves begins that day rather unexpectedly. As the growthrate for T increases more than that for B, the T : B-proportion too begins to rise the same day. In cross 8 (4 × 14) the F2-generation was quite uniform for W, a frequencycurve with one mode only in the same class as those of the parents. The B-values gave a bimodal curve, so that a monohybrid segregation (128 : 44) could be assumed; one of the modes agrees on the value of strain 4, which also had been shown by the F1-generation, the other was the same as that of strain 14. Thus the same growthfactor G2 may be assumed for both strains 14 and 4, while an additional factor seems to be present in strain 4, giving an intensified growth of the B-part. In the same manner the T-values in F1 corresponded to those of strain 4; the F2-generation segregated (again with a few exceptional intermediates) in two types like the parents (125 : 3 : 44). The fundamental factor G2 appears here too as the growthfactor for the T-parts, to which in strain 4 an additional factor T2 is added, whose activity however is of a more intense nature than the T1-factor mentioned above. For the T1 in strain 11 produced an increase from 130 to 200; the T2-factor in this strain 4 causes an extension from 200 to 465. This assumption that here really an additional factor T2 of greater intensity is at work, is corroborated by the segregation in the reciprocal crosses 7 and 7*a* (4 × 11) in which F2 formed a curve with three modes; for strain 11 the formula G1T1 had been found and therefore a formula G2T1 for strain 4 does not suit the experimental data as it would produce a bimodal curve. In consequence of the threemodal curves (86 : 57 : 11 and 92 : 65 : 12) we must assume for strain 4 the formula G2T2, which causes a dihybrid segregation when crossed with strain 11 G1T1. As might be expected, the cross 4 × 11 for W segregated in a monohybrid way (G2 × G1, 242 : 77) and for B according to the dihybrid 9 : 6 : 1-scheme (G2B1 × G1b, 180 : 121 : 18).

The other cross, in which strain 4 was used together with one of the above mentioned types, viz. cross 5 and 5a (4 × 6) confirmed these assumptions on the understanding that the stronger differences between the parents gave four clear modes for the T-values, but besides some intermediate individuals (G4 t × G2T2, 156 : 7 : 54 : 2 : 60 : 5 : 15). Here the T-values in the F1 and in the main group of F2 exceeded the G4-value

in a remarkable way; the length of these T-parts was even more than 9 cm.

Strain 8 was characterised by growth-curves which for T entirely agreed on that of the G3-type ; for W and B they seemed like those of the G4-type, though the final values exceeded somewhat the dimensions of strain 6. Only two crosses with a strain thusfar discussed have been made. Cross 41 (5 × 8) showed for T a uniform F2-generation, which corroborates the G3-assumption, both W and B segregated in a monohybrid way, which is in accordance with the formulas for strain 8 G3W1 and G3B1. (G3w × G3W1, 84 : 25 ; G3b × G3B1, 83 : 26).

In cross 36 (8 × 11) the three dimensions were the same in the F1-individuals and a little greater than those of the G4-values. In F2-generations the three dimensions again behaved in the same way; their segregations were of a dihybrid nature (60 : 23 : 23 : 8; 58 : 26 : 24 : 6; 62 : 20 : 25 : 7). Above the formula for strain 11 was determined as G1wbT1; the segregations in F2 can be interpreted as in cross 41 by the assumption that strain 8 possesses the growthfactor G3 and besides the additional factors W1 and B1. Thus the segregations for W, B and T are all the same G3A1 : G3a : G1A1 : G1a. It might seem a difficulty that the numbers for G3 (A1 or a) and G1 (A1 or a) are not the same for these three dimensions (W83 : 31; B84 : 30; T82 : 32), but this may probably be explained by the fact that G3 phenotypically is the same as G2A1, while G1A1 is like G2, so that we are entitled to assume the presence of a few of these exceptional types among the phenotypes G3 and G1A1. Then the real segregations would be for W 60 : 20 : 3 : 2 : 21 : 8; for B 58 : 22 : 4 : 1 : 23 : 6 and for T 62 : 18 : 2 : 3 : 22 : 7. In this way we find 80 G3 : 5 G2 : 29 G1.

The next strain 1 produced a normal G3-curve for B; the W-curve was like that of strain 8, while T showed a very steep curve, which has not been observed in any other strain. An analogous reasoning as given above, would lead to the supposition that here the factor G3 is responsible for the growth in the three dimensions ; to this fundamental factor W1 may be added and possibly the very intensive factor T2, which had been observed in strain 4. Strain 1 was used a.o. for the following crosses: 19 (1 × 6), 39 (1 × 5), 1 (1 × 14), 21 (1 × 11), 20 (1 × 4) and 40 (8 × 1), so that many opportunities were given to test this supposition.

Cross 39 (1 × 5) gave a monohybrid segregation for W (113 : 40, G3W1 × G3w); no segregation for B (G3b × G3b) and again a monohybrid segregation for T with a few intermediates (G3T2 × G3t, 110 : 5 : 38).

Cross 40 (8 × 1) produced in F2 for W a uniform group (G3W1 × G3W1), for B a monohybrid segregation (G3B1 × G3b. 106 : 32) and for T again a monohybrid but now with some intermediates (G3T2 × G3t). In cross 19 (1 × 6) the W-segregation was of a dihybrid nature (G3W1 × G4w, 83 : 26 : 27 : 10); the B-curve showed two modes (G3b × G4b, 110 : 36), while T segregated into four groups and a few

intermediates (G3T2 × G4t, 80 : 26 : 6 : 24 : 10). The W-families in cross 1 (1 × 14) gave in F2 a trimodal curve (G3W1 × G2w, 78 : 45 : 6), B a monohybrid segregation (G3b × G2b, 10 : 29) and T again a dihybrid one and intermediates (G3T2 × G2t, 69 : 21 : 5 : 28 : 6). From cross 20 (1 × 4) the F2-generation segregated for W according to two factors (G3W1 × G2w, 91 : 59 : 8), for B dihybrid (G3b × G2B1, 94 : 53 : 11) and for T monohybrid (G3T2 × G2T2, 122 : 36). Further gave cross 21 (1 × 11) a dihybrid W-segregation (G3W1 × G1w, 70 : 27 : 26 : 9). monohybrid B-families with intermediates (G3b × G1b, 95 : 4 : 33) while T segregated in a dihybrid way with intermediates (G3T2 × G1T1, 68 : 4 : 27 : 26 : 7). The numerical proportions in these crosses all confirm the formulas added.

The remaining strain 2 was used in these crosses : 2 (2 × 4), 13 (2 × 1) 15 (2 × 11), 16 (2 × 14), 37 (8 × 2) and 38 (6 × 2). The clearest results have been given by cross 38 : B and T were entirely uniform in F2, while W showed a monohybrid segregation (101 : 28). On this ground we could assume that the fundamental factor for the three dimensions was the same as that in strain 6 (G4); the width was increased by the aid of the W1-factor. This preliminary formula has found full corroboration in the other crosses: Cross 37 (8 × 2) gave a W-segregation of 94 : 31 (G3W1 × G4W1), a dihybrid B (G3B1 × G4b, 69 : 24 : 25 : 7) and a monohybrid T (G3w × G4w, 94 : 31). Cross 13 (2 × 1) gave for W 118 : 38 (G4W1 × G3W1), for B 118 : 38 (G4b × G3b) and for T 91 : 27 : 2 : 3 : 25 : 8 (G4t × G3T2, with intermeditates). The other crosses 2 (2 × 4, G4W1bt × G2wB1T2), 15 (G4W1bt × C1wbT1) and 16 (2 × 14, G4W1bt × G3wbt) behaved entirely as might be expected on the basis of these formulas. The numbers are given in the table. The growthcurves are also in keeping with these formulas; for B and T they are identical with those of strain 6, while W from the seventh day on is increased by the influence of W1.

Summarizing the data given above, we are allowed to put forth the following conclusions:

1. Fundamentally the three dimensions W, B and T are influenced by a common cause, which may found on a series of quadruple allelomorphs: G4, G3, G2 and G1. These four allelomorphs are able to produce respectively about 410, 295, 200 and 130 unities ($^1/_{10}$ mm.). The segregations between these multiple allelomorphs suit to the monohybrid scheme, a few exceptional intermediate types reserved, when no directly subsequent parental factors are concerned.

2. Each of these dimensions can be influenced by separate additional factors, which cause a definite increase of the values produced by the G-factors. The activity of these W1-, B1-, and T1-factors is about the same; beside T1 still another T2-factor has been observed, which is allelomorph to T1 but whose intensity is much greater. Crosses of T2

with the recessive t produce again exceptional intermediates. For B and W no other additional factors have been found.

3. The values caused by the possible genotypic combinations are about the following (A = additional factor):

	A2	A1	a
G4	925	630	420
G3	660	440	295
G2	465	300	200
G1	300	200	130

4. The genotypic combinations G3a, G2A1 and G1A2 are phenotypically alike; the same can be said of G2a and G1A1.

5. The growthcurves for each of these dimensions gave a full affirmation of the results obtained from the crosses; those for the G-factors show an entirely regular behaviour, while the additional factors seem to begin their activity on the seventh day of observation.

6. It is possible to draw up for each of the eight strains used, a genotypic formula, which may aid to give an interpretation of the results of crosses as also of the growthcurves: *6* G4wbt; *5* G3wbt; *14* G2wbt; *11* G1wbT1; *4* G2wB1T2; *8* G3W1B1t; *1* G3W1bT2 and *2* G4W1bt.

Of course the final values, mentioned above, which may be gained by the plant as a consequence of its genotypic constitution, can be determined by approximation only, even if they have been calculated from a large number of measurements. Nevertheless it is a striking fact, that in the proportions between the values a certain regularity cannot be denied. The ratios of two succeeding values vary very little; they amount to about 1.50 (G4 : G3 = 420 : 295 = 1.42; G3 : G2 = 295 : 200 = 1.47; and G2 : G1 = 200 : 130 = 1.54). In view of this undeniable regularity, a certain proportionality of the quantities of G-factors is not at all improbable. In the same way a comparison between the A2- and A1-activities leads to the same conclusion:

G4A2 : G4A1 = 925 : 630 = 1.47. G4A1 : G4a = 630 : 420 = 1.50.
G3A2 : G3A1 = 660 : 440 = 1.50. G3A1 : G3a = 440 : 295 = 1.49.
G2A2 : G2A1 = 465 : 300 = 1.55. G2A1 : G2a = 300 : 200 = 1.50.
G1A2 : G1A1 = 300 : 200 = 1.50. G1A1 : G1a = 200 : 130 = 1.54.

All differences in intensity thus result into a proportion of about 1.50 : 1 and therefore they may be properly ascribed to differences in quantity.

The exceptional intermediate types have an essential theoretical

importance, as they form another valuable argument in favour of the quantitative nature of these factors. In a recent paper (1929. p. 486), EAST and YARNELL have pointed out some difficulties in the theory of quantitative genes. They say: "When any of two allelomorphs of a series are brought together in an organism, only forms containing these allelomorphs are recovered. If the quantitative theory of heredity is correct, one should expect to recover other forms." This statement now appears to be groundless; other, intermediate, forms have been found in the above data.

However the interpretation of the origin of these intermediate types is open to various opinions. In my paper of 1926 I have drawn the attention to CORRENS' explanation of analogous phenomena in a variegated Shepherds purse (CORRENS, 1919), and I still am inclined to adhere to this explanation. But I must admit that the remarkable observations on modifying factors in Drosophila, which are located in other loci of the same chromosome, give perhaps right to other assumptions (e.g. HERSH, 1929). It may be that the extreme G4-factor fundamentally is composed of four unities, which are rather strongly linked, but which can show crossover-phenomena, when its allelomorph possesses only one or two of these unities. For the time being it will remain impossible to decide between these two opinions; the crucial test would be the determination of the locus or loci in which this or these growth-factor(s) are located and sofar I donot dispose of any suitable data. Linkage between the G-factor(s) and any other gene has not yet been observed; the T-factor(s) seem to be linked to the factor for green seedcoat, but the crossovervalues between this factor and the T2- or T1-factor(s) have given very slight differences only, so that their significance is very doubtful.

The above data and conclusions clearly show that the problems of quantitative inheritance are not yet solved; therefore I cannot agree with the words of O. TEDIN (1929. p. 11): "The general rules for the inheritance of these characters are well known and do not seem to offer, at present, any unsolved problems of great fundamental importance. And any investigator of the fundamental laws of factorial inheritance does well in avoiding the multiple factors, for reasons only too readily understood."

On the contrary, in my opinion, the problems of quantitative inheritance have become of vital interest in genetics; they have been neglected too long and their study is quite essential for obtaining real notions on the nature of genes.

LITERATURE CITED.

CORRENS, C., 1919. Vererbungsversuche mit buntblättrigen Sippen. I. Capsella Bursa pastoris albovariabilis und chlorina. (Sitzungsber. preuss. Akad. d. Wiss. **34**. 1919. p. 585—610).

EAST, E. M. and S. H. YARNELL, 1929. Studies on self-sterility, VIII. Selfsterility allelomorphs. (Genetics. **14**. 1929. p. 455—487).

EMERSON, R. A. and E. M. EAST, 1913. The inheritance of quantitative characters in maize. (Nebraska Agr. Exp. Stat. Res. Bull. **2**. 1913. 120 pp.).

EMERSON, R. A., 1916. A genetic study of plantheight in Phaseolus vulgaris. (Nebraska Agr. Exp. Stat. Res. Bull. **7**. 1916. 73 pp.).

HERSH, A. H. 1929. The effect of different sections of the X-chromosome upon bar eye in Drosophila melanogaster. (Amer. Natur. **63**. 1929. p. 378—382).

NILSSON—EHLE, H. 1909—1911. Kreuzungsstudien an Hafer und Weizen. I—II. (Lunds Univ. Aarsskr. N. F. Afd. 2. V. N⁰. 2. 122 pp. und VII. N⁰. 6. 83 pp.)

REED, H. S. 1919—1924. Various papers in Proc. nat. Acad. Sc. Wash. **5**. 1919; **7**, 1921; **9**, 1923. Amer. Natur. **55**, 1921; **58**, 1924. Journ. Gen. Phys. **2**, 1920. Journ. Agr. Res. **21**, 1921. Amer. Journ. Bot. **7**, 1920. a.o.

———————— 1924. The nature of growth. (Amer. Natur. **58**. 1924. p. 337—349),

ROBERTSON, T. B. 1923. The chemical basis of growth and senescence. (Philadelphia—London, Lippincott, 1923. 389 pp.).

SANDE BAKHUYZEN, H. L. v. D. 1926. Growth and growth formulas in plants. (Science. **64**. 1926. p. 653—654).

SANDE BAKHUYZEN, H. L. v. D. and E. L. ALSBERG 1927. The growth curve in annual plants. (Physiol. Reviews. **7**. 1927. p. 151—187).

SIRKS, M. J. 1926. Multiple allelomorphs versus multiple factors. (Proc. Int. Congr. Plantsciences. Ithaca, 1926. I. p. 803—814. publ. 1929).

TEDIN, O. 1929. On the result of simultaneous gametic and environmental correlations in a segregating population. (Hereditas. **12**. 1929. p. 11—16).

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Mathematics. — *Skew Correlation between Three and More Variables,* III. By Prof. M. J. VAN UVEN. (Communicated by Prof. A. A. NIJLAND).

(Communicated at the meeting of May 25, 1929).

II. *Skew Correlation between n variables.*

We now pass on to the treatment of the correlation problem with n variables. So we start with an n-dimensional frequency distribution of the n variables $x_1, x_2, \ldots, x_n$.

The successive limits of the ν_a classes of x_a may be designated by $x_a(0)$, $x_a(1)$, $x_a(2), \ldots, x_a(k_a), \ldots, x_a(\nu_a)$.

So $x_a(k_a)$ is the upper limit of the k_a^{th} class. That x_a is found within the k_a^{th} class, means therefore that $x_a(k_a-1) < x_a < x_a(k_a)$.

The frequency of the set of the n classes $x_a(k_a-1) < x_a < x_a(k_a)$, $a = 1, \ldots, n$ may be denoted by $Y(k_1, k_2, \ldots, k_n)$.

Then the frequency distribution is described by the statement:

For $Y(k_1, k_2, \ldots k_n)$ sets $(x_1, x_2, \ldots, x_n)$ is found $x_a(k_a-1) < x_a < x_a(k_a)$, $(a = 1, \ldots, n.)$ } **A**

So the total number of observed sets amounts to

$$N = \sum_{i_1=1}^{\nu_1} \sum_{i_2=1}^{\nu_2} \ldots \sum_{i_n=1}^{\nu_n} Y(i_1, i_2, \ldots, i_n) \quad . \quad . \quad . \quad . \quad . \quad 1^{(n)}$$

and the *relative* frequencies (empirical probabilities) are

$$y(k_1, k_2, \ldots, k_n) = \frac{Y(k_1, k_2, \ldots, k_n)}{N} \quad . \quad . \quad . \quad . \quad 2^{(n)}$$

We shall, also in studying n-dimensional correlation, try to interpret the given frequency distribution **A** by a *linear* correlation between n (unimodular) variables t_a $(a = 1, \ldots n)$, each of which is a function of only one, or of more, of the original variables x_a.

The infinitesimal probability formula for this n unimodular variables runs:

$$dW = \frac{\sqrt{\Delta}}{\sqrt{\pi^n}} e^{-f} dt_1 . dt_2 \ldots dt_n , \quad . \quad . \quad . \quad 52 \; [=4^{(n)}]$$

where

$$f \equiv \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} \lambda_{\alpha\beta} t_\alpha t_\beta , \quad . \quad . \quad . \quad . \quad . \quad . \quad 53 \; [=5^{(n)}]$$

being

$$\lambda_{\alpha\alpha} = 1, \quad \lambda_{\alpha\beta} = \lambda_{\beta\alpha}, \quad \lambda_{\alpha\beta}^2 < 1, \quad \Delta = |\lambda_{\alpha\beta}| \text{ with minors } \Delta_{\alpha\beta} \; . \; 6^{(n)}$$

Contrarily to the method followed in the case of three variables, we shall, this time, effectuate the division of f algebraically.

We first introduce the "polar" magnitudes γ_{ab}, determined by

$$\gamma_{ab} = \Lambda_{ab} : \sqrt{\Lambda_{aa}\,\Lambda_{bb}} \qquad (\gamma_{ba} = \gamma_{ab}) \quad . \quad . \quad . \quad . \quad \textbf{13ter (n)}$$

These γ_{ab} are the total coefficients of correlation of the linearly correlated variables $t_1, t_2, \ldots, t_n$.

The relation between the magnitudes λ_{ab} and γ_{ab} is mutual, that is to say, we have also:

$$\lambda_{ab} = \Gamma_{ab} : \sqrt{\Gamma_{aa}\,\Gamma_{bb}}, \quad . \quad . \quad . \quad . \quad \textbf{54} \ [=\textbf{13ter (n)}]$$

where Γ_{ab} is the minor (algebraic complement) of the element γ_{ab} in the determinant $\Gamma = |\gamma_{\alpha\beta}|$.

So, we may, in virtue of **54**, write for f:

$$f = \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} \{\Gamma_{\alpha\beta}\, t_\alpha\, t_\beta : \sqrt{\Gamma_{\alpha\alpha}\,\Gamma_{\beta\beta}}\} \quad . \quad . \quad . \quad . \quad . \quad \textbf{53bis}$$

We shall now divide f into a sum of n squares, in such a way, that the first term contains only t_1, the second only t_1 and $t_2, \ldots$, the last all the t_a. From

$$f_n \equiv \frac{\partial f}{\partial t_n} = 2 \sum_{\alpha=1}^{n} \lambda_{\alpha n}\, t_\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \textbf{55}$$

ensues, that in $\left(\frac{f_n}{2}\right)^2$ appear all the terms of f, which contain t_n. Introducing pro forma $\lambda_{nn}\,(=1)$ as a factor, and putting:

$$\frac{f_n}{2} : \sqrt{\lambda_{nn}} = \zeta_{n;\,n-1,\ldots,1}, \quad . \quad . \quad . \quad \textbf{56 (n; n—1, \ldots, 1)}$$

that is to say, defining $\zeta_{n;\,n-1,\ldots,1}$ by

$$\zeta_{n;\,n-1,\ldots,1} = \left\{\sum_{\alpha=1}^{n} \lambda_{\alpha n}\, t_\alpha\right\} : \sqrt{\lambda_{nn}} = \left(\frac{1}{\Gamma_{nn}}\right)^{1/2} \cdot \sum_{\alpha=1}^{n} \{\Gamma_{\alpha n}\, t_\alpha : \sqrt{\Gamma_{\alpha\alpha}}\} \ \textbf{57 (n; n—1, \ldots, 1)}$$

we may write for the form f:

$$f = \zeta^2_{n;\,n-1,\ldots,1} + f', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \textbf{58}$$

where

$$f' \equiv \sum_{\alpha=1}^{n-1} \sum_{\beta=1}^{n-1} \lambda'_{\alpha\beta}\, t_\alpha\, t_\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \textbf{53'}$$

contains only terms with $t_1, \ldots, t_{n-1}$.

For the coefficient λ'_{ab} we have

$$\lambda'_{ab} = \lambda_{ab} - \frac{\lambda_{an}\,\lambda_{nb}}{\lambda_{nn}} = \begin{vmatrix} \lambda_{ab}, & \lambda_{an} \\ \lambda_{nb}, & \lambda_{nn} \end{vmatrix} : \lambda_{nn} \quad . \quad . \quad . \quad . \quad . \quad \textbf{54'}$$

If we wish to express the coefficients λ'_{ab} in the (total) coefficients of correlation γ_{ab}, we find:

$$\lambda'_{ab} = \begin{vmatrix} \Gamma_{ab}, & \Gamma_{an} \\ \Gamma_{nb}, & \Gamma_{nn} \end{vmatrix} : (\Gamma_{nn} \cdot \sqrt{\Gamma_{aa}\,\Gamma_{bb}}).$$

Now we have in general:

$$\begin{vmatrix} \Gamma_{ab}, & \Gamma_{aj} \\ \Gamma_{ib}, & \Gamma_{ij} \end{vmatrix} = \Gamma . \Gamma_{ab|ij} , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 59$$

where $\Gamma_{ab|ij}$ represents the algebraic complement of $\begin{vmatrix} \gamma_{ab}, & \gamma_{aj} \\ \gamma_{ib}, & \gamma_{ij} \end{vmatrix}$. Thus $\Gamma_{ab|ij}$ is equal to $(-1)^{a+b+i+j}$ times the rest-determinant of order $n-2$, which arises by dropping the rows a and i and the columns b and j in Γ.

So we find for λ'_{ab}:

$$\lambda'_{ab} = \Gamma . \Gamma_{ab|nn} : (\Gamma_{nn} . \surd \Gamma_{aa} \Gamma_{bb}) \quad . \quad . \quad . \quad . \quad . \quad 54^{bis}$$

and for f':

$$\left.\begin{aligned} f' &= \left\{ \sum_{\alpha=1}^{n-1} \sum_{\beta=1}^{n-1} \begin{vmatrix} \lambda_{\alpha\beta}, & \lambda_{\alpha n} \\ \lambda_{n\beta}, & \lambda_{nn} \end{vmatrix} t_\alpha t_\beta \right\} : \lambda_{nn} \\ &= (\Gamma : \Gamma_{nn}) . \sum_{\alpha=1}^{n-1} \sum_{\beta=1}^{n-1} \{ \Gamma_{\alpha\beta|nn} t_\alpha t_\beta : \surd \Gamma_{\alpha\alpha} \Gamma_{\beta\beta} \} \end{aligned}\right\} . \quad . \quad . \quad . \quad 60'$$

We now consider:

$$f'_{n-1} \equiv \frac{\partial f'}{\partial t_{n-1}} = 2 \sum_{\alpha=1}^{n-1} \lambda'_{\alpha, n-1} t_\alpha ,$$

observing that in $\left(\frac{f'_{n-1}}{2}\right)^2 : \lambda'_{n-1, n-1}$ appear all the terms of f', which contain t_{n-1}.

Putting:

$$\frac{f'_{n-1}}{2} : \surd \lambda'_{n-1, n-1} = \zeta_{n-1; n-2, \ldots, 1} , \quad . \quad 56\,(n-1; n-2, \ldots, 1)$$

we obtain

$$\zeta_{n-1; n-2, \ldots, 1} = \left\{ \sum_{\alpha=1}^{n-1} \lambda'_{\alpha, n-1} t_\alpha \right\} : \surd \lambda'_{n-1, n-1} =$$

$$= \left\{ \sum_{\alpha=1}^{n-1} \begin{vmatrix} \lambda_{\alpha, n-1}, & \lambda_{\alpha n} \\ \lambda_{n, n-1}, & \lambda_{nn} \end{vmatrix} t_\alpha \right\} : \surd \left(\lambda_{nn} . \begin{vmatrix} \lambda_{n-1, n-1}, & \lambda_{n-1, n} \\ \lambda_{n, n-1} & , \lambda_{nn} \end{vmatrix} \right)$$

$$= \left\{ \Gamma : (\Gamma_{nn} \surd \Gamma_{n-1, n-1}) \right\} . \sum_{\alpha=1}^{n-1} \left\{ \Gamma_{\alpha, n-1|nn} t_\alpha : \surd \Gamma_{\alpha\alpha} \right\} : \surd \left\{ \Gamma . \Gamma_{n-1, n-1|nn} : \Gamma_{nn} . \Gamma_{n-1, n-1} \right\}$$

$$= \left\{ \Gamma : \Gamma_{nn} . \Gamma_{n-1, n-1|nn} \right\}^{1/2} . \sum_{\alpha=1}^{n-1} \left\{ \Gamma_{\alpha, n-1|nn} t_\alpha : \surd \Gamma_{\alpha\alpha} \right\} ,$$

hence

$$\left.\begin{aligned} \zeta_{n-1; n-2, \ldots, 1} &= \left\{ \sum_{\alpha=1}^{n-1} \begin{vmatrix} \lambda_{\alpha, n-1}, & \lambda_{\alpha n} \\ \lambda_{n, n-1}, & \lambda_{nn} \end{vmatrix} t_\alpha \right\} : \surd \left(\lambda_{nn} . \begin{vmatrix} \lambda_{n-1, n-1}, & \lambda_{n-1, n} \\ \lambda_{n, n-1} & , \lambda_{nn} \end{vmatrix} \right) \\ &= \left\{ \Gamma : (\Gamma_{nn} . \Gamma_{n-1, n-1|nn}) \right\}^{1/2} . \sum_{\alpha=1}^{n-1} \left\{ \Gamma_{\alpha, n-1|nn} t_\alpha : \surd \Gamma_{\alpha\alpha} \right\} \end{aligned}\right\} 57\,(n-1; n-2, \ldots, 1)$$

Thus we may write for f':

$$f' = \zeta^2_{n-1; n-2, \ldots, 1} + f'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 58'$$

where

$$f'' \equiv \sum_{\alpha=1}^{n-2} \sum_{\beta=1}^{n-2} \lambda''_{\alpha\beta}\, t_\alpha\, t_\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 53''$$

contains only terms with $t_1, \ldots, t_{n-2}$.

For the coefficient λ''_{ab} we have:

$$\lambda''_{ab} = \lambda'_{ab} - \frac{\lambda'_{a,n-1}\,\lambda'_{n-1,b}}{\lambda'_{n-1,n-1}} = \begin{vmatrix} \lambda_{ab} & \lambda_{a,n-1} & \lambda_{an} \\ \lambda_{n-1,b} & \lambda_{n-1,n-1} & \lambda_{n-1,n} \\ \lambda_{nb} & \lambda_{n,n-1} & \lambda_{nn} \end{vmatrix} : \begin{vmatrix} \lambda_{n-1,n-1} & \lambda_{n-1,n} \\ \lambda_{n,n-1} & \lambda_{nn} \end{vmatrix} \quad 54''$$

or, by **54**,

$$\lambda''_{ab} = \left\{ \begin{vmatrix} \Gamma_{ab} & \Gamma_{a,n-1} & \Gamma_{an} \\ \Gamma_{n-1,b} & \Gamma_{n-1,n-1} & \Gamma_{n-1,n} \\ \Gamma_{nb} & \Gamma_{n,n-1} & \Gamma_{nn} \end{vmatrix} : (\Gamma_{n-1,n-1} \cdot \Gamma_{nn} \sqrt{\Gamma_{aa}\,\Gamma_{bb}}) \right\} :$$

$$: \left\{ \begin{vmatrix} \Gamma_{n-1,n-1} & \Gamma_{n-1,n} \\ \Gamma_{n,n-1} & \Gamma_{nn} \end{vmatrix} : (\Gamma_{n-1,n-1} \cdot \Gamma_{nn}) \right\}$$

$$= \begin{vmatrix} \Gamma_{ab} & \Gamma_{a,n-1} & \Gamma_{an} \\ \Gamma_{n-1,b} & \Gamma_{n-1,n-1} & \Gamma_{n-1,n} \\ \Gamma_{nb} & \Gamma_{n,n-1} & \Gamma_{nn} \end{vmatrix} : \left(\begin{vmatrix} \Gamma_{n-1,n-1} & \Gamma_{n-1,n} \\ \Gamma_{n,n-1} & \Gamma_{nn} \end{vmatrix} \cdot \sqrt{\Gamma_{aa}\,\Gamma_{bb}} \right).$$

Now we have in general:

$$\begin{vmatrix} \Gamma_{ab} & \Gamma_{aj} & \Gamma_{al} \\ \Gamma_{ib} & \Gamma_{ij} & \Gamma_{il} \\ \Gamma_{kb} & \Gamma_{kj} & \Gamma_{kl} \end{vmatrix} = \Gamma^2 \cdot \Gamma_{ab|ij|kl}, \quad . \quad . \quad . \quad . \quad . \quad . \quad 59'$$

where $\Gamma_{ab|ij|kl}$ represents the algebraic complement of $\begin{vmatrix} \gamma_{ab} & \gamma_{aj} & \gamma_{al} \\ \gamma_{ib} & \gamma_{ij} & \gamma_{il} \\ \gamma_{kb} & \gamma_{kj} & \gamma_{kl} \end{vmatrix}$ in Γ, thus being equal to $(-1)^{a+b+i+j+k+l}$ times the rest-determinant, we obtain by dropping the rows a, i, k and the columns b, j, l of Γ.

So we may transform the expression **54''** for λ''_{ab} into:

$$\lambda''_{ab} = \Gamma^2 \cdot \Gamma_{ab|n-1,n-1|nn} : (\Gamma \cdot \Gamma_{n-1,n-1|nn} \cdot \sqrt{\Gamma_{aa}\,\Gamma_{bb}}),$$

or

$$\lambda''_{ab} = \Gamma \cdot \Gamma_{ab|n-1,n-1|nn} : (\Gamma_{n-1,n-1|nn} \cdot \sqrt{\Gamma_{aa}\,\Gamma_{bb}}) \quad . \quad . \quad . \quad 54''\text{bis}$$

Hence we find for f'' the expression:

$$f'' = \left\{ \sum_{\alpha=1}^{n-2} \sum_{\beta=1}^{n-2} \begin{vmatrix} \lambda_{\alpha\beta} & \lambda_{\alpha,n-1} & \lambda_{\alpha n} \\ \lambda_{n-1,\beta} & \lambda_{n-1,n-1} & \lambda_{n-1,n} \\ \lambda_{n\beta} & \lambda_{n,n-1} & \lambda_{nn} \end{vmatrix} t_\alpha\, t_\beta \right\} : \begin{vmatrix} \lambda_{n-1,n-1} & \lambda_{n-1,n} \\ \lambda_{n,n-1} & \lambda_{nn} \end{vmatrix} \Bigg\} \quad 60''$$

$$= (\Gamma : \Gamma_{n-1,n-1|nn}) \cdot \sum_{\alpha=1}^{n-2} \sum_{\beta=1}^{n-2} \{ \Gamma_{\alpha\beta|n-1,n-1|nn}\, t_\alpha\, t_\beta : \sqrt{\Gamma_{\alpha\alpha}\,\Gamma_{\beta\beta}} \}$$

Proceeding in the same manner, we obtain first

$$\zeta_{n-2;\,n-3,\ldots,1} = \tfrac{1}{2}\frac{\partial f''}{\partial t_{n-2}} : \sqrt{\lambda''_{n-2,\,n-2}} = \Big\{ \sum_{\alpha=1}^{n-2} \lambda''_{\alpha,\,n-2}\, t_\alpha \Big\} : \sqrt{\lambda''_{n-2,\,n-2}}$$

$$= \left\{ \sum_{\alpha=1}^{n-2} \begin{vmatrix} \lambda_{\alpha,\,n-2} & \lambda_{\alpha,\,n-1} & \lambda_{\alpha n} \\ \lambda_{n-1,\,n-2} & \lambda_{n-1,\,n-1} & \lambda_{n-1,\,n} \\ \lambda_{n,\,n-2} & \lambda_{n,\,n-1} & \lambda_{nn} \end{vmatrix} t_\alpha \right\} :$$

$$: \sqrt{\left(\begin{vmatrix} \lambda_{n-1,\,n-1} & \lambda_{n-1,\,n} \\ \lambda_{n,\,n-1} & \lambda_{nn} \end{vmatrix} \cdot \begin{vmatrix} \lambda_{n-2,\,n-2} & \lambda_{n-2,\,n-1} & \lambda_{n-2,\,n} \\ \lambda_{n-1,\,n-2} & \lambda_{n-1,\,n-1} & \lambda_{n-1,\,n} \\ \lambda_{n,\,n-2} & \lambda_{n,\,n-1} & \lambda_{nn} \end{vmatrix} \right)}$$

$$= (\Gamma : \Gamma_{n-1,\,n-1|nn}) \cdot \sum_{\alpha=1}^{n-2} \{ \Gamma_{\alpha,\,n-2|n-1,\,n-1|nn}\, t_\alpha : \sqrt{(\Gamma_{\alpha\alpha} \cdot \Gamma_{n-2,\,n-2})} \} :$$

$$: \sqrt{\{ \Gamma \cdot \Gamma_{n-2,\,n-2|n-1,\,n-1|nn} : (\Gamma_{n-1,\,n-1|nn} \cdot \Gamma_{n-2,\,n-2}) \}}$$

$$= \{ \Gamma : (\Gamma_{n-1,\,n-1|nn} \cdot \Gamma_{n-2,\,n-2|n-1,\,n-1|nn}) \}^{1/3} \cdot \sum_{\alpha=1}^{n-2} \{ \Gamma_{\alpha,\,n-2|n-1,\,n-1|nn}\, t_\alpha : \sqrt{\Gamma_{\alpha\alpha}} \},$$

whence:

$$\left.\begin{aligned} \zeta_{n-2;\,n-3,\ldots,1} &= \left\{ \sum_{\alpha=1}^{n-2} \begin{vmatrix} \lambda_{\alpha,\,n-2} & \lambda_{\alpha,\,n-1} & \lambda_{\alpha n} \\ \lambda_{n-1,\,n-2} & \lambda_{n-1,\,n-1} & \lambda_{n-1,\,n} \\ \lambda_{n,\,n-2} & \lambda_{n,\,n-1} & \lambda_{nn} \end{vmatrix} t_\alpha \right\} : \\ &: \sqrt{\left(\begin{vmatrix} \lambda_{n-1,\,n-1} & \lambda_{n-1,\,n} \\ \lambda_{n,\,n-1} & \lambda_{nn} \end{vmatrix} \cdot \begin{vmatrix} \lambda_{n-2,\,n-2} & \lambda_{n-2,\,n-1} & \lambda_{n-2,\,n} \\ \lambda_{n-1,\,n-2} & \lambda_{n-1,\,n-1} & \lambda_{n-1,\,n} \\ \lambda_{n,\,n-2} & \lambda_{n,\,n-1} & \lambda_{nn} \end{vmatrix} \right)} \\ &= \{ \Gamma : (\Gamma_{n-1,\,n-1|nn} \cdot \Gamma_{n-2,\,n-2|n-1,\,n-1|nn}) \}^{1/3} \cdot \\ &\quad \cdot \sum_{\alpha=1}^{n-2} \{ \Gamma_{\alpha,\,n-2|n-1,\,n-1|nn}\, t_\alpha : \sqrt{\Gamma_{\alpha\alpha}} \} \end{aligned}\right\} \mathbf{57\ (n-2;\,n-3,\ldots,1)}$$

Continuing the division of f according to the same principle, we arrive, en passant, at

$$\left.\begin{aligned} f^{(j)} \equiv \sum_{\alpha=1}^{n-j} \sum_{\beta=1}^{n-j} \lambda^{(j)}_{\alpha\beta}\, t_\alpha\, t_\beta &= \left\{ \sum_{\alpha=1}^{n-j} \sum_{\beta=1}^{n-j} \begin{vmatrix} \lambda_{\alpha\beta} & \lambda_{\alpha,\,n-j+1} & \cdots & \lambda_{\alpha n} \\ \lambda_{n-j+1,\,\beta} & \lambda_{n-j+1,\,n-j+1} & \cdots & \lambda_{n-j+1,\,n} \\ \vdots & \vdots & & \vdots \\ \lambda_{n\beta} & \lambda_{n,\,n-j+1} & \cdots & \lambda_{nn} \end{vmatrix} t_\alpha\, t_\beta \right\} : \\ &: \begin{vmatrix} \lambda_{n-j+1,\,n-j+1} & \cdots & \lambda_{n-j+1,\,n} \\ \vdots & & \vdots \\ \lambda_{n,\,n-j+1} & \cdots & \lambda_{nn} \end{vmatrix} \\ &= (\Gamma : \Gamma_{n-j+1,\,n-j+1|\ldots|nn}) \cdot \sum_{\alpha=1}^{n-j} \sum_{\beta=1}^{n-j} \{ \Gamma_{\alpha\beta|n-j+1,\,n-j+1|\ldots|nn}\, t_\alpha\, t_\beta : \sqrt{\Gamma_{\alpha\alpha}\, \Gamma_{\beta\beta}} \} \end{aligned}\right\} \mathbf{60(j)}$$

Putting:

$$n - j = b$$

and

$$\Gamma_{b+1,\,b+1|b+2,\,b+2|\ldots|nn} = \begin{vmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1b} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2b} \\ \vdots & \vdots & & \vdots \\ \gamma_{b1} & \gamma_{b2} & \cdots & \gamma_{bb} \end{vmatrix} = \overset{1,\ldots,b}{\Gamma},$$

whence

$$\overset{1,\ldots,b-1}{\Gamma} = \overset{1,\ldots,b}{\Gamma_{bb}},$$

we may write for **60**$^{(j)}$:

$$f^{(n-b)} = \left\{ \sum_{\alpha=1}^{b} \sum_{\beta=1}^{b} \begin{vmatrix} \lambda_{\alpha\beta} & \lambda_{\alpha,\,b+1} & \ldots, & \lambda_{\alpha n} \\ \lambda_{b+1,\,\beta} & \lambda_{b+1,\,b+1} & \ldots, & \lambda_{b+1,\,n} \\ \vdots & \vdots & & \vdots \\ \lambda_{n,\beta} & \lambda_{n,\,b+1} & \ldots, & \lambda_{nn} \end{vmatrix} t_\alpha\, t_\beta \right\} : \begin{vmatrix} \lambda_{b+1,\,b+1} & \ldots, & \lambda_{b+1,\,n} \\ \vdots & & \vdots \\ \lambda_{n,\,b+1} & \ldots, & \lambda_{nn} \end{vmatrix} \Bigg\} \mathbf{60^{(n-b)}}$$

$$= (\overset{1,\ldots,b}{\Gamma} : \overset{1,\ldots,b}{\Gamma}) \cdot \sum_{\alpha=1}^{b} \sum_{\beta=1}^{b} \{ \Gamma_{\alpha\beta}\, t_\alpha\, t_\beta : \surd \Gamma_{\alpha\alpha}\, \Gamma_{\beta\beta} \}.$$

Likewise we obtain for the function ζ:

$$\zeta_{n-j;\,n-j-1,\,\ldots,\,1} = \left\{ \sum_{\alpha=1}^{n-j} \begin{vmatrix} \lambda_{\alpha,\,n-j} & \ldots, & \lambda_{\alpha n} \\ \lambda_{n-j+1,\,n-j} & \ldots, & \lambda_{n-j+1,\,n} \\ \vdots & & \vdots \\ \lambda_{n,\,n-j} & \ldots, & \lambda_{nn} \end{vmatrix} t_\alpha \right\} :$$

$$: \surd \left(\begin{vmatrix} \lambda_{n-j+1,\,n-j+1} & \ldots, & \lambda_{n-j+1,\,n} \\ \vdots & & \vdots \\ \lambda_{n,\,n-j+1} & \ldots, & \lambda_{nn} \end{vmatrix} \cdot \begin{vmatrix} \lambda_{n-j,\,n-j} & \ldots, & \lambda_{n-j,\,n} \\ \vdots & & \vdots \\ \lambda_{n,\,n-j} & \ldots, & \lambda_{nn} \end{vmatrix} \right) \Bigg\} \mathbf{57\ (n-j;\,n-j-1,\ldots,1)}$$

$$= \{ \Gamma : (\Gamma_{n-j+1,\,n-j+1|\ldots|nn} \cdot \Gamma_{n-j,\,n-j|\ldots|nn}) \}^{1/2} \cdot$$

$$\cdot \sum_{\alpha=1}^{n-j} \{ \Gamma_{\alpha,\,n-j|n-j+1,\,n-j+1|\ldots|nn}\, t_\alpha : \surd \Gamma_{\alpha\alpha} \},$$

or

$$\zeta_{b;\,b-1,\ldots,1} = \left\{ \sum_{\alpha=1}^{b} \begin{vmatrix} \lambda_{\alpha b} & \ldots, & \lambda_{\alpha n} \\ \lambda_{b+1,\,b} & \ldots, & \lambda_{b+1,\,n} \\ \vdots & & \vdots \\ \lambda_{nb} & \ldots, & \lambda_{nn} \end{vmatrix} t_\alpha \right\} :$$

$$: \surd \left(\begin{vmatrix} \lambda_{b+1,\,b+1} & \ldots, & \lambda_{b+1,\,n} \\ \vdots & & \vdots \\ \lambda_{n,\,b+1} & \ldots, & \lambda_{nn} \end{vmatrix} \cdot \begin{vmatrix} \lambda_{bb} & \ldots, & \lambda_{bn} \\ \vdots & & \vdots \\ \lambda_{nb} & \ldots, & \lambda_{nn} \end{vmatrix} \right) \Bigg\} \mathbf{57\ (b;\,b-1,\ldots,1)}$$

$$= \{ \overset{1,\ldots,b}{\Gamma} : (\overset{1,\ldots,b}{\Gamma} \cdot \overset{1,\ldots,b}{\Gamma_{bb}}) \}^{1/2} \cdot \sum_{\alpha=1}^{b} \{ \overset{1,\ldots,b}{\Gamma_{\alpha b}}\, t_\alpha : \surd \Gamma_{\alpha\alpha} \}.$$

Moreover we have:

$$f^{(j)} = \zeta^2_{n-j;\,n-j-1,\ldots,1} + f^{(j+1)}, \quad \ldots\ldots \quad \mathbf{58^{(j)}}$$

or

$$f^{(n-b)} = \zeta^2_{b;\,b-1,\ldots,1} + f^{(n-b+1)}, \quad \ldots\ldots \quad \mathbf{58^{(n-b)}}$$

and

$$f = \zeta^2_{n;\,n-1,\ldots,1} + \zeta^2_{n-1;\,n-2,\ldots,1} + \ldots + \zeta^2_{n-j;\,n-j-1,\ldots,1} + f^{(j+1)}, \quad \mathbf{61}$$

or

$$f = \zeta^2_{n;\,n-1,\ldots,1} + \zeta^2_{n-1;\,n-2,\ldots,1} + \ldots + \zeta^2_{b;\,b-1,\ldots,1} + f^{(n-b+1)} \quad \mathbf{61^{bis}}$$

At last we obtain successively:

$$f^{(n-3)} \equiv \sum_{\alpha=1}^{3}\sum_{\beta=1}^{3} \lambda_{\alpha\beta}^{(n-3)}\, t_\alpha\, t_\beta = \left\{ \sum_{\alpha=1}^{3}\sum_{\beta=1}^{3} \begin{vmatrix} \lambda_{\alpha,3}, & \lambda_{\alpha 4}, \ldots, & \lambda_{\alpha n} \\ \lambda_{43}, & \lambda_{44}, \ldots, & \lambda_{4n} \\ \vdots & \vdots & \vdots \\ \lambda_{n3}, & \lambda_{n4}, \ldots, & \lambda_{nn} \end{vmatrix} t_\alpha\, t_\beta \right\} : \begin{vmatrix} \lambda_{44}, \ldots, & \lambda_{4n} \\ \vdots & \vdots \\ \lambda_{n4}, \ldots, & \lambda_{nn} \end{vmatrix}$$

$$= (\Gamma : \overset{1,2,3}{\Gamma}) \cdot \sum_{\alpha=1}^{3}\sum_{\beta=1}^{3} \{ \overset{1,2,3}{\Gamma}\, t_\alpha\, t_\beta : \surd \Gamma_{\alpha\alpha}\, \Gamma_{\beta\beta} \}, \qquad \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \mathbf{60^{(n-3)}}$$

$$\zeta_{3;21} = \left\{ \sum_{\alpha=1}^{3} \begin{vmatrix} \lambda_{\alpha 3}, & \ldots, & \lambda_{\alpha n} \\ \lambda_{43}, & \ldots, & \lambda_{4n} \\ \vdots & \vdots & \vdots \\ \lambda_{n3}, & \ldots, & \lambda_{nn} \end{vmatrix} t_\alpha \right\} : \surd\left(\begin{vmatrix} \lambda_{44}, \ldots, & \lambda_{4n} \\ \vdots & \vdots \\ \lambda_{n4}, \ldots, & \lambda_{nn} \end{vmatrix} \cdot \begin{vmatrix} \lambda_{33}, \ldots, & \lambda_{3n} \\ \vdots & \vdots \\ \lambda_{n3}, \ldots, & \lambda_{nn} \end{vmatrix} \right)$$

$$= \{ \Gamma : (\overset{1,2,3}{\Gamma} \cdot \overset{1,2,3}{\Gamma_{33}}) \}^{1/2} \cdot \sum_{\alpha=1}^{3} \{ \overset{1,2,3}{\Gamma_{\alpha 3}}\, t_\alpha : \surd \Gamma_{\alpha\alpha} \}, \qquad \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \mathbf{57\ (3;\ 21)}$$

or, in virtue of

$$\begin{vmatrix} \lambda_{33}, \ldots, & \lambda_{3n} \\ \vdots & \vdots \\ \lambda_{n3}, \ldots, & \lambda_{nn} \end{vmatrix} = \Delta_{11|22}, \text{ etc.},$$

$$f^{(n-3)} = (\Delta_{22|33} \cdot t_1^2 - 2\,\Delta_{21|33} \cdot t_1\, t_2 + \Delta_{11|33} t_2^2 + 2\,\Delta_{31|22} \cdot t_1\, t_3 - 2\,\Delta_{11|32} \cdot t_2 t_3 + \Delta_{11|22} \cdot t_3^2) : \Delta_{11|22|33}, \qquad \mathbf{60^{(n-3)\ bis}}$$

$$\zeta_{3;21} = (\Delta_{22|31} \cdot t_1 - \Delta_{11|32} \cdot t_2 + \Delta_{11|22}\, t_3) : \surd(\Delta_{11|22|33} \cdot \Delta_{11|22}); \qquad \mathbf{57^{(3;\ 21)\ bis}}$$

$$f^{(n-2)} = (\Delta_{22} \cdot t_1^2 - 2\,\Delta_{12} \cdot t_1\, t_2 + \Delta_{11} \cdot t_2^2) : \Delta_{11|22} = (\Gamma : \overset{1,2}{\Gamma}) \cdot (\gamma_{22} \cdot t_1^2 : \Gamma_{11} - 2\gamma_{12} \cdot t_1\, t_2 : \surd \Gamma_{11}\, \Gamma_{22} + \gamma_{11} \cdot t_2^2 : \Gamma_{22}), \qquad \mathbf{60^{(n-2)\ bis}}$$

$$\zeta_{2;1} = (-\Delta_{21} \cdot t_1 + \Delta_{11} \cdot t_2) : \surd(\Delta_{11|22} \cdot \Delta_{11}) = \{ \Gamma : (\overset{1,2}{\Gamma} \cdot \overset{1,2}{\Gamma_{22}}) \}^{1/2} \cdot (-\gamma_{21} \cdot t_1 : \surd \Gamma_{11} + \gamma_{11} \cdot t_2 : \surd \Gamma_{22}); \qquad \mathbf{57^{(2;\ 1)\ bis}}$$

$$f^{(n-1)} = \Delta \cdot t_1^2 : \Delta_{11} = (\Gamma : \overset{1}{\Gamma}) \cdot (t_1^2 : \Gamma_{11}) = \Gamma \cdot t_1^2 : \Gamma_{11} \quad . \quad . \quad . \qquad \mathbf{60^{(n-1)\ bis}}$$

$$\zeta_1 = \Delta \cdot t_1 : \surd(\Delta_{11} \cdot \Delta) = (\Delta : \Delta_{11})^{1/2} \cdot t_1 = (\Gamma : \Gamma_{11})^{1/2} \cdot t_1; \qquad \mathbf{57^{(1)\ bis}}$$

whence

$$f^{(n-1)} = \zeta_1^2 \quad . \quad . \quad . \quad . \quad . \quad . \qquad \mathbf{60^{(n-1)\ ter}}$$

We may, in designating the variable $\zeta_{n;\,n-1,\ldots,1}$, content ourselves with only one subscript. Hence we write

$$\zeta_{n;\,n-1,\ldots,1} = Z_n .$$

Moreover, replacing the symbol ζ_1 by the symbol z_1:

$$\zeta_1 = z_1,$$

we may summarize the expressions found above in the following way:

$$
\left.
\begin{aligned}
z_1 &= (\Lambda : \Lambda_{11})^{1/2} \,.\, t_1 = (\Gamma : \Gamma_{11})^{1/2} \,.\, t_1 \\
\zeta_{2;1} &= (-\Lambda_{21} \,.\, t_1 + \Lambda_{11} \,.\, t_2) : \sqrt{(\Lambda_{11|22} \,.\, \Lambda_{11})} = \\
&= \{\Gamma : (\Gamma.1)\}^{1/2} \,.\, (-\gamma_{21}\, t_1 : \sqrt{\Gamma_{11}} + t_2 : \sqrt{\Gamma_{22}}) \\
\vdots \qquad & \qquad \vdots \\
\zeta_{b;\,b-1,\ldots,1} &= \frac{\sum_{\alpha=1}^{b} \begin{vmatrix} \lambda_{\alpha b} & \ldots, & \lambda_{\alpha n} \\ \lambda_{b+1,\,b} & \ldots, & \lambda_{b+1,\,n} \\ \lambda_{n,\,b} & \ldots, & \lambda_{nn} \end{vmatrix} t_\alpha}{\sqrt{\left(\begin{vmatrix} \lambda_{b+1,\,b+1} & \ldots, & \lambda_{b+1,\,n} \\ \lambda_{n,\,b+1} & \ldots, & \lambda_{nn} \end{vmatrix} \,.\, \begin{vmatrix} \lambda_{bb} & \ldots, & \lambda_{bn} \\ \lambda_{nb} & \ldots, & \lambda_{nn} \end{vmatrix} \right)}} = \\
&= \{\Gamma : (\Gamma^{1\ldots b}_{1\ldots b} \,.\, \Gamma_{bb})\}^{1/2} \,.\, \sum_{\alpha=1}^{b} \{\Gamma^{1\ldots b}_{\alpha b}\, t_\alpha : \sqrt{\Gamma_{\alpha\alpha}}\} \\
\zeta_{n-1;\,n-2,\ldots,1} &= \frac{\sum_{\alpha=1}^{n-1} \begin{vmatrix} \lambda_{\alpha,\,n-1} & \lambda_{\alpha n} \\ \lambda_{n,\,n-1} & \lambda_{nn} \end{vmatrix} t_\alpha}{\sqrt{\left(\lambda_{nn} \,.\, \begin{vmatrix} \lambda_{n-1,\,n-1} & \lambda_{n-1,\,n} \\ \lambda_{n,\,n-1} & \lambda_{nn} \end{vmatrix} \right)}} = \\
&= \{\Gamma : (\Gamma_{nn} \,.\, \Gamma_{nn|n-1,\,n-1})\}^{1/2} \,.\, \sum_{\alpha=1}^{n-1} \{\Gamma_{\alpha,\,n-1|nn}\, t_\alpha : \sqrt{\Gamma_{\alpha\alpha}}\}
\end{aligned}
\right\} \quad 57\ ^{1})
$$

$$
Z_n = \frac{\sum_{\alpha=1}^{n} \lambda_{\alpha n}\, t_\alpha}{\sqrt{\lambda_{nn}}} = \{\Gamma : (\Gamma \,.\, \Gamma_{nn})\}^{1/2} \,.\, \sum_{\alpha=1}^{n} \{\Gamma_{\alpha n}\, t_\alpha : \sqrt{\Gamma_{\alpha\alpha}}\} = (1 : \Gamma_{nn})^{1/2} \,.\, \sum_{\alpha=1}^{n} \{\Gamma_{\alpha n}\, t_\alpha : \sqrt{\Gamma_{\alpha\alpha}}\}
$$

[1]) We may write the formulae **57** still in another form, making use of the so-called "partial mean deviations" and "partial coefficients of regression."

For the *total* mean deviation τ_b of the *unimodular* variable t_b holds: $\tau_b^2 = \Gamma_{bb} : 2\Gamma$.

The partial mean deviation of order $b-1$: $\tau_{b;\,b-1,\ldots,1}$ of t_b ($t_1, \ldots, t_{b-1}$ being kept constant) satisfies the relation:

$$
\tau^2_{b;\,b-1,\ldots,1} = (\Gamma^{1,\ldots,b}_{1,\ldots,b} : \Gamma_{bb})\, \tau_b^2 = (\Gamma^{1,\ldots,b}_{1,\ldots,b} \,.\, \Gamma_{bb}) : (2\, \Gamma^{1,\ldots,b}_{1,\ldots,b} \,.\, \Gamma_{bb}).
$$

For the partial coefficient of regression of order $b-2$: $\varrho_{b|\alpha;\,b-1,\ldots,\underset{\alpha}{\downarrow},\ldots,1}$ of t_b with respect to t_α (keeping constant $t_1, \ldots, t_{b-1}$, except t_α) we have:

$$
\varrho_{b|\alpha;\,b-1,\ldots,\underset{\alpha}{\downarrow},\ldots,1} = (-\Gamma^{1,\ldots,b\ \ 1,\ldots,b}_{\alpha b} : \Gamma_{bb}) \,.\, (\tau_b : \tau_\alpha) = (-\Gamma^{1,\ldots,b\ \ 1,\ldots,b}_{\alpha b} : \Gamma_{bb}) \,.\, \sqrt{(\Gamma_{bb} : \Gamma_{\alpha\alpha})}.
$$

So the equation **57 (b; b−1, ..., 1)** may be transformed as follows:

$$
\begin{aligned}
\zeta_{b;\,b-1,\ldots,1} &= \{\Gamma : (\Gamma^{1,\ldots,b\ \ 1,\ldots,b}_{1,\ldots,b} \,.\, \Gamma_{bb})\}^{1/2} \,.\, \{\Gamma^{1,\ldots,b\ \ 1,\ldots,b}_{bb}\, t_b : \sqrt{\Gamma_{bb}} + \sum_{\alpha=1}^{b-1} (\Gamma^{1,\ldots,b\ \ 1,\ldots,b}_{\alpha b}\, t_\alpha : \sqrt{\Gamma_{\alpha\alpha}})\} \\
&= \{\Gamma^{1,\ldots,b\ \ 1,\ldots,b}_{bb} : (\Gamma^{1,\ldots,b\ \ 1,\ldots,b}_{1,\ldots,b} \,.\, \Gamma_{bb})\}^{1/2} \,.\, \{t_b + \sum_{\alpha=1}^{b-1} (\Gamma^{1,\ldots,b\ \ 1,\ldots,b}_{\alpha b} : \Gamma_{bb}) \,.\, t_\alpha \,.\, \sqrt{(\Gamma_{bb} : \Gamma_{\alpha\alpha})}\} \\
&= \frac{t_b - \sum_{\alpha=1}^{b-1} \varrho_{b|\alpha;\,b-1,\ldots,\underset{\alpha}{\downarrow},\ldots,1} \,.\, t_\alpha}{\sqrt{2} \,.\, \tau_{b;\,b-1,\ldots,1}}.
\end{aligned}
$$

When, in the numerator of this latter fraction, we replace t_b by the *mean* value which t_b takes if the values of $t_1, \ldots, t_{b-1}$ are prescribed, then we obtain, by annulling

Putting in each formula:

$$t_\alpha = (\Gamma_{\alpha\alpha} : \Gamma)^{1/2} \cdot z_\alpha ,$$

then **57** passes into:

$$\left.\begin{aligned}
z_1 &= z_1 \\
\zeta_{2;1} &= (-\gamma_{21}\, z_1 + z_2) : \sqrt{\overset{1.2}{\Gamma}} \\
\zeta_{3;21} &= (\overset{3}{\underset{\alpha=1}{\Sigma}} \overset{1.2,3}{\Gamma_{\alpha 3}} \cdot z_\alpha) : \sqrt{(\overset{1.2,3}{\Gamma} \cdot \overset{1.2,3}{\Gamma_{33}})} \\
\zeta_{b;\,b-1,\ldots,1} &= (\overset{b}{\underset{\alpha=1}{\Sigma}} \overset{1,\ldots,b}{\Gamma_{\alpha b}} \cdot z_\alpha) : \sqrt{(\overset{1,\ldots,b}{\Gamma} \cdot \overset{1,\ldots,b}{\Gamma_{bb}})} = \\
&= (\overset{b}{\underset{\alpha=1}{\Sigma}} \Gamma_{\alpha b|b+1,\,b+1|\ldots|nn} \cdot z_\alpha) : \sqrt{(\Gamma_{b+1,\,b+1|\ldots|nn} \cdot \Gamma_{bb|\ldots|nn})} \\
\zeta_{n-1;\,n-2,\ldots,1} &= (\overset{n-1}{\underset{\alpha=1}{\Sigma}} \Gamma_{\alpha,\,n-1|nn} \cdot z_\alpha) : \sqrt{(\Gamma_{nn} \cdot \Gamma_{n-1,\,n-1|nn})} \\
Z_n &= (\overset{n}{\underset{\alpha=1}{\Sigma}} \Gamma_{\alpha n} \cdot z_\alpha) : \sqrt{(\Gamma \cdot \Gamma_{nn})}.
\end{aligned}\right\} \text{57bis}$$

Moreover we put:

$$f^{(n-b)} = \zeta^2_{b;\,b-1,\ldots,1} + \zeta^2_{b-1;\,b-2,\ldots,1} + \ldots + \zeta^2_{2;1} + z^2_1 = q^2_{12\ldots b-1,\,b} \quad . \quad 62$$

$$f = Z^2_n + \zeta^2_{n-1;\,n-2,\ldots,1} + \ldots + \zeta^2_{2;1} + z^2_1 = q^2_{12\ldots,\,n-1,\,n} = r^2 \quad . \quad 63$$

In virtue of the way in which we have built up the magnitudes $q^2_{12\ldots b-1.b}$, they are independent of the arrangement of the subscripts $1, 2, \ldots, b$, thus independent of the order in which we have constructed the functions z_α, $\zeta_{\beta;\alpha}$, $\zeta_{\gamma;\beta\alpha}$, $\ldots$ $\alpha, \beta, \gamma, \ldots = 1, \ldots, b$.

So we have, for instance,

$$q^2_{ab} = q^2_{ba} = z^2_a + \zeta^2_{b;a} = z^2_b + \zeta^2_{a;b} \quad . \quad . \quad . \quad . \quad . \quad . \quad 64$$

Moreover from

$$\zeta_{b;a} = \frac{-\gamma_{ba}\, z_a + z_b}{\sqrt{1-\gamma^2_{ba}}} \quad \text{and} \quad \zeta_{a;b} = \frac{-\gamma_{ab}\, z_b + z_a}{\sqrt{1-\gamma^2_{ab}}} \quad (\gamma_{ba} = \gamma_{ab}) \quad . \quad 65$$

follows the relation

$$q^2_{ab} = \frac{z^2_a - 2\gamma_{ab}\, z_a\, z_b + z^2_b}{1-\gamma^2_{ab}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 66$$

We have further

$$z_a\, z_b - \zeta_{b;a}\, \zeta_{a;b} = \frac{(1-\gamma^2_{ab})\, z_a\, z_b - \gamma^2_{ab}\, z_a\, z_b + \gamma_{ab}\, z^2_a + \gamma_{ab}\, z^2_b - z_a\, z_b}{1-\gamma^2_{ab}} = \gamma_{ab} \cdot q^2_{ab},$$

---

this numerator, the so-called "equation of regression" of order $b-1$ for t_b, with respect to $t_1, \ldots, t_{b-1}$.

The (second) equation **57 (2)** passes into $\zeta_{2;1} = \frac{t_2 - \varrho_{2|1} \cdot t_1}{\sqrt{2} \cdot r_2}$, where r_2 is the *total* mean deviation of t_2, and $\varrho_{2|1}$ the *total* coefficient of regression of t_2 with respect to t_1.

whence

$$\gamma_{ab} = \frac{z_a\, z_b - \zeta_{b;a}\, \zeta_{a;b}}{q^2_{ab}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 67$$

So γ_{ab} is built up of $z_a, z_b, \zeta_{b;a}, \zeta_{a;b}$.
We also have

$$z_a\, \zeta_{a;b} + z_b\, \zeta_{b;a} = \frac{-\gamma_{ab}\, z_a\, z_b + z_a^2 - \gamma_{ab}\, z_a\, z_b + z_b^2}{\sqrt{1-\gamma^2_{ab}}} = \sqrt{1-\gamma^2_{ab}} \cdot q^2_{ab},$$

whence

$$\sqrt{1-\gamma^2_{ab}} = \frac{z_a\, \zeta_{a;b} + z_b\, \zeta_{b;a}}{q^2_{ab}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 68$$

and

$$M_{ab} = \frac{\sqrt{1-\gamma^2_{ab}}}{\gamma_{ab}} = \frac{z_a\, \zeta_{a;b} + z_b\, \zeta_{b;a}}{z_a\, z_b - \zeta_{b;a}\, \zeta_{a;b}} \quad . \quad . \quad . \quad . \quad . \quad 69$$

Putting in **60(n−b)** $t_\alpha = (\Gamma_{\alpha\alpha} : \Gamma)^{1/2} \cdot z_\alpha$, $t_\beta = (\Gamma_{\beta\beta} : \Gamma)^{1/2} \cdot z_\beta$, $f^{(n-b)} = q^2_{12\ldots b}$ we obtain

$$q^2_{12\ldots b} = \{ \sum_{\alpha=1}^{b} \sum_{\beta=1}^{b} {}^{1,\ldots,b}\Gamma_{\alpha\beta}\, z_\alpha\, z_\beta \} : {}^{1,\ldots,b}\Gamma \quad . \quad . \quad . \quad . \quad . \quad 70\text{(b)}$$

Since

$$\frac{\partial\,(z_1, \zeta_{2;1}, \ldots, Z_n)}{\partial\,(t_1, t_2, \ldots, t_n)} = \sqrt{\Delta},$$

we may put the infinitesimal probability formula into the form:

$$\left.\begin{aligned} dW = \frac{1}{\sqrt{\pi^n}}\, e^{-(z_1^2 + \zeta_{2;1}^2 + \ldots + Z_n^2)}\, dz_1 \cdot d\zeta_{2;1} \ldots dZ_n = \\ = d\Theta\{z_1\} \cdot d\Theta\{\zeta_{2;1}\} \ldots d\Theta\{Z_n\}. \end{aligned}\right\} \quad . \quad . \quad 71$$

The probability of the set $t_1, t_2, \ldots, t_b$ (with ranges $dt_1, dt_2, \ldots, dt_b$), $t_{b+1}, \ldots, t_n$ being arbitrary, is (theoretically) expressed by

$$d_{(b+1,\ldots,n)} W = \frac{1}{\sqrt{\pi^n}} \int_{t_{b+1}=-\infty}^{+\infty} \ldots \int_{t_n=-\infty}^{+\infty} dW = d\Theta\{z_1\} \cdot d\Theta\{\zeta_{2;1}\} \ldots d\Theta\{\zeta_{b;\,b-1,\ldots,1}\}.$$

Thus the probability of $[t_b < t_b\,(k_b), t_{b+1}, \ldots t_n$ arbitr., *being given:* $t_1\,(k_1 - 1) < t_1 < t_1\,(k_1), \ldots t_{b-1}\,(k_{b-1} - 1) < t_{b-1} < t_{b-1}\,(k_{b-1})]$ has for its *theoretical* value:

$$\sigma_{b;\,b-1,\ldots,1}\,(k_1 - \tfrac{1}{2}, \ldots, k_{b-1} - \tfrac{1}{2}, k_b) = \Theta\{\zeta_{b;\,b-1,\ldots,1}\,(k_1 - \tfrac{1}{2}, \ldots, k_{b-1} - \tfrac{1}{2}, k_b)\}, \quad 72$$

where

$$\left.\begin{aligned} \zeta_{b;\,b-1,\ldots,1}\,(k - \tfrac{1}{2}, \ldots, k_{b-1} - \tfrac{1}{2}, k_b) = \\ = \{ \sum_{\alpha=1}^{b-1} {}^{1,\ldots,b}\Gamma_{\alpha b} \cdot z_\alpha\,(k_\alpha - \tfrac{1}{2}) + {}^{1,\ldots,b}\Gamma_{bb} \cdot z_b\,(k_b) \} : \sqrt{({}^{1,\ldots,b}\Gamma \cdot {}^{1,\ldots,b}\Gamma_{bb})}. \end{aligned}\right\} \quad 73$$

Supposing each variable t_a to be an (ever increasing) function of only x_a, the probability of $t_a < t_a(k_a)$ is equal to that of $x_a < x_a(k_a)$; hence we have likewise:

Probability of $[t_b < t_b(k_b), t_{b+1}, \ldots, t_n$ arbitr., *being given*: $t_a(k_a - 1) < t_a < t_a(k_a)$ for $a = 1, \ldots, b-1] =$ Probability of $[x_b < x_b(k_b), x_{b+1}, \ldots x_n$ arbitr. *being given*: $x_a(k_a - 1) < x_a < x_a(k_a)$ for $a = 1, \ldots, b-1]$.

We next consider the *empirical* probabilities:

The probability of $[x_1 < x_1(k_1), x_2, \ldots x_n$ being arbitrary] has for its *empirical* value:

$$s_1(k_1) = \sum_{i_1=1}^{k_1} \sum_{i_2=1}^{\nu_2} \ldots \sum_{i_n=1}^{\nu_n} y(i_1, i_2, \ldots, i_n) = \frac{\sum_{i_1=1}^{k_1} \sum_{i_2=1}^{\nu_2} \ldots \sum_{i_n=1}^{\nu_n} Y(i_1, i_2, \ldots, i_n)}{N}; \quad 72(1)$$

the probability of $[x_2 < x_2(k_2), x_3, \ldots x_n$ arbitr., *being given*: $x_1(k_1 - 1) < x_1 < x_1(k_1)]$ has the *empirical* value:

$$\left.\begin{aligned} \sigma_{2;1}(k_1 - \tfrac{1}{2}, k_2) &= \frac{\sum_{i_2=1}^{k_2} \sum_{i_3=1}^{\nu_3} \ldots \sum_{i_n=1}^{\nu_n} y(k_1, i_2, i_3, \ldots, i_n)}{\sum_{i_2=1}^{\nu_2} \sum_{i_3=1}^{\nu_3} \ldots \sum_{i_n=1}^{\nu_n} y(k_1, i_2, i_3, \ldots, i_n)} = \\ &= \frac{\sum_{i_2=1}^{k_2} \sum_{i_3=1}^{\nu_3} \ldots \sum_{i_n=1}^{\nu_n} Y(k_1, i_2, i_3, \ldots, i_n)}{\sum_{i_2=1}^{\nu_2} \sum_{i_3=1}^{\nu_3} \ldots \sum_{i_n=1}^{\nu_n} Y(k_1, i_2, i_3, \ldots, i_n)}; \end{aligned}\right\} 72(2;1)$$

the probability of $[x_b < x_b(k_b), x_{b+1}, \ldots, x_n$ arbitr., *being given*: $x_a(k_a - 1) < x_a < x_a(k_a)$ for $a = 1, \ldots, b-1]$ has the *empirical* value:

$$\left.\begin{aligned} &\sigma_{b;\,b-1,\ldots,1}(k_1 - \tfrac{1}{2}, \ldots, k_{b-1} - \tfrac{1}{2}, k_b) = \\ &= \frac{\sum_{i_b=1}^{k_b} \sum_{i_{b+1}=1}^{\nu_{b+1}} \ldots \sum_{i_n=1}^{\nu_n} Y(k_1, \ldots, k_{b-1}, i_b, i_{b+1}, \ldots, i_n)}{\sum_{i_b=1}^{\nu_b} \sum_{i_{b+1}=1}^{\nu_{b+1}} \ldots \sum_{i_n=1}^{\nu_n} Y(k_1, \ldots, k_{b-1}, i_b, i_{b+1}, \ldots, i_n)} \end{aligned}\right\} 72(b;\,b-1),\ldots,1$$

the probability of $[x_n < x_n(k_n)$, *being given*: $x_a(k_a - 1) < x_a < x_a(k_a)$ for $a = 1, \ldots, n-1]$ has the *empirical* value:

$$\left.\begin{aligned} &\sigma_{n;\,n-1,\ldots,1}(k_1 - \tfrac{1}{2}, \ldots, k_{n-1} - \tfrac{1}{2}, k_n) = \\ &= \frac{\sum_{i_n=1}^{k_n} Y(k_1, \ldots, k_{n-1}, i_n)}{\sum_{i_n=1}^{\nu_n} Y(k_1, \ldots, k_{n-1}, i_n)} = S_n(k_1 - \tfrac{1}{2}, \ldots, k_{n-1} - \tfrac{1}{2}, k_n). \end{aligned}\right\} 72(n;\,n-1,\ldots,1)$$

By equalizing the empirical values of these probabilities to their corresponding theoretical values (**72**), we obtain:

$$\left.\begin{aligned} s_1(k_1) &= \frac{\sum_{i_1=1}^{k_1}\sum_{i_2=1}^{\nu_2}\dots\sum_{i_n=1}^{\nu_n} Y(i_1, i_2, \dots, i_n)}{\sum_{i_1=1}^{\nu_1}\sum_{i_2=1}^{\nu_2}\dots\sum_{i_n=1}^{\nu_n} Y(i_1, i_2, \dots, i_n)} = \Theta\{z_1(k_1)\}, \\ \text{where} \quad z_1(k_1) &= (\Gamma : \Gamma_{11})^{1/2} \cdot t_1(k_1); \end{aligned}\right\} \quad \mathbf{B}^{(1)}$$

$$\left.\begin{aligned} \sigma_{2;1}(k_1 - \tfrac{1}{2}, k_2) &= \frac{\sum_{i_2=1}^{k_2}\sum_{i_3=1}^{\nu_3}\dots\sum_{i_n=1}^{\nu_n} Y(k_1, i_2, i_3, \dots, i_n)}{\sum_{i_2=1}^{\nu_2}\sum_{i_3=1}^{\nu_3}\dots\sum_{i_n=1}^{\nu_n} Y(k_1, i_2, i_3, \dots, i_n)} = \Theta\{\zeta_{2;1}(k_1 - \tfrac{1}{2}, k_2)\}, \\ \text{where} \quad \zeta_{2;1}(k_1 - \tfrac{1}{2}, k_2) &= \frac{-\gamma_{21} \cdot z_1(k_1 - \frac{1}{2}) + z_2(k_2)}{\sqrt{(1 - \gamma_{21}^2)}}; \end{aligned}\right\} \quad \mathbf{B}^{(2;1)}$$

$$\left.\begin{aligned} &\sigma_{b;\,b-1,\dots,1}(k_1 - \tfrac{1}{2}, \dots, k_{b-1} - \tfrac{1}{2}, k_b) = \\ &= \frac{\sum_{i_b=1}^{k_b}\sum_{i_{b+1}=1}^{\nu_{b+1}}\dots\sum_{i_n=1}^{\nu_n} Y(k_1, \dots, k_{b-1}, i_b, i_{b+1}, \dots, i_n)}{\sum_{i_b=1}^{\nu_b}\sum_{i_{b+1}=1}^{\nu_{b+1}}\dots\sum_{i_n=1}^{\nu_n} Y(k_1, \dots, k_{b-1}, i_b, i_{b+1}, \dots, i_n)} = \\ &= \Theta\{\zeta_{b;\,b-1,\dots,1}(k_1 - \tfrac{1}{2}, \dots, k_{b-1} - \tfrac{1}{2}, k_b)\}. \end{aligned}\right\} \quad \mathbf{B}^{(b;\,b-1,\dots,1)}$$

where

$$\left.\begin{aligned} &\zeta_{b;\,b-1,\dots,1}(k_1 - \tfrac{1}{2}, \dots, k_{b-1} - \tfrac{1}{2}, k_b) = \\ &= \frac{\sum_{\alpha=1}^{b-1} \overset{1,\dots,b}{\Gamma}_{\alpha b} \cdot z_\alpha(k_\alpha - \frac{1}{2}) + \overset{1,\dots,b}{\Gamma}_{bb} \cdot z_b(k_b)}{\sqrt{(\overset{1,\dots,b}{\Gamma} \cdot \overset{1,\dots,b}{\Gamma}_{bb})}}; \end{aligned}\right\}$$

$$\left.\begin{aligned} S_n(k_1 - \tfrac{1}{2}, \dots, k_{n-1} - \tfrac{1}{2}, k_n) &= \frac{\sum_{i_n=1}^{k_n} Y(k_1, \dots, k_{n-1}, i_n)}{\sum_{i_n=1}^{\nu_n} Y(k_1, \dots, k_{n-1}, i_n)} = \\ &= \Theta\{Z_n(k_1 - \tfrac{1}{2}, \dots, k_{n-1} - \tfrac{1}{2}, k_n)\}, \end{aligned}\right\} \quad \mathbf{B}^{(n;\,n-1,\dots,1)}$$

where

$$Z_n(k_1 - \tfrac{1}{2}, \dots, k_{n-1} - \tfrac{1}{2}, k_n) = \frac{\sum_{\alpha=1}^{n-1} \Gamma_{\alpha n} \cdot z_\alpha(k_\alpha - \frac{1}{2}) + \Gamma_{nn} \cdot z_n(k_n)}{\sqrt{(\Gamma \cdot \Gamma_{nn})}}.$$

The equation $\mathbf{B}^{(1)}$ associates a value $z_1(k_1)$ with each value $x_1(k_1)$. Applying the same method to x_a, we join to each value $x_a(k_a)$ the corresponding value $z_a(k_a)$. So we determine empirically the n functions

$z_a \{x_a\}$ $(a = 1, \ldots, n)$. The equation **B(2,1)** now associates a value $\zeta_{2;1}(k_1 - \frac{1}{2}, k_2)$ with each set $x_1(k_1 - \frac{1}{2}), x_2(k_2)$. So we obtain for each set $z_1(k_1 - \frac{1}{2}), z_2(k_2)$ a value of the magnitude $\zeta_{2;1}$, which, in this way, is constructed also as a function of z_1 and z_2. Here $z_1(k_1 - \frac{1}{2})$ is the mean value between $z_1(k_1 - 1)$ and $z_1(k_1)$ which must represent the class k_1.

All the functions $\zeta_{b;a}$ having been determined in a similar way, we use them, together with the function z_a, to compute from them the total coefficients of correlation γ_{ab} (by means of **69**).

Putting

$$z_a^2 + \zeta_{b;a}^2 = q_{b;a}^2, \quad \ldots\ldots\ldots \quad \mathbf{C}$$

it must appear — provided the supposition $t_a \{x_a\}$ be right — that, for each combination $x_a(k_a)$, $x_b(k_b)$ $(k_a = 1, \ldots, \nu_a, \quad k_b = 1, \ldots, \nu_b)$ we have:

$$q_{a;b} = q_{b;a} (= q_{ab}) \ldots\ldots\ldots \quad \mathbf{Ia}$$

Following the same way as in the case of three variables, we put:

$$z_a z_b - \zeta_{b;a}\zeta_{a;b} = A_{ab} \quad , \quad z_a \zeta_{a;b} + z_b \zeta_{b;a} = B_{ab}. \quad \ldots \quad \mathbf{D}$$

Then

$$A_{ab}^2 + B_{ab}^2 = (z_a^2 + \zeta_{b;a}^2)(z_b^2 + \zeta_{a;b}^2) = q_{b;a}^2 \cdot q_{a;b}^2$$

Introducing the arc ω_{ab}, defined by

$$\operatorname{tg} \omega_{ab} = \frac{B_{ab}}{A_{ab}} \left(\frac{\cos \omega_{ab}}{A_{ab}} > 0 \right),$$

we obtain:

$$\cos \omega_{ab} = \frac{A_{ab}}{q_{b;a} \cdot q_{a;b}}, \quad \sin \omega_{ab} = \frac{B_{ab}}{q_{b;a} \cdot q_{a;b}}, \quad \operatorname{tg} \omega_{ab} = \frac{B_{ab}}{A_{ab}} \left(\frac{\cos \omega_{ab}}{A_{ab}} > 0 \right) \quad \mathbf{E}$$

If **Ia** is really satisfied, we have (see **67**, **68**, **69**)

$$\left.\begin{aligned} \gamma_{ab} = \cos \omega_{ab} = \frac{A_{ab}}{q_{ab}^2}, \quad \sqrt{1-\gamma_{ab}^2} = \sin \omega_{ab} = \frac{B_{ab}}{q_{ab}^2}, \\ M_{ab} = \frac{\sqrt{1-\gamma_{ab}^2}}{\gamma_{ab}} = \operatorname{tg} \omega_{ab} = \frac{B_{ab}}{A_{ab}}. \end{aligned}\right\} \quad \mathbf{Ebis}$$

Using the equation **70(b)** we can compute $q_{12\ldots b}^2$ as a function of z_α and $\gamma_{\alpha\beta}$ $(\alpha, \beta = 1, \ldots, b)$ or, which comes to the same, of the magnitudes $z_\alpha, \zeta_{\beta;\alpha}$ $(\alpha, \beta = 1, \ldots, b)$. In particular we obtain for $f = q_{12\ldots n}^2 = r^2$:

$$r^2 = \frac{\sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} \Gamma_{\alpha\beta} z_\alpha z_\beta}{\Gamma} \quad \ldots\ldots\ldots \quad \mathbf{70(n)}$$

By means of **58(n−b)**, or

$$q_{12\ldots b}^2 = \zeta_{b;b-1,\ldots,1}^2 + q_{12\ldots,b-1}^2,$$

we can control $\zeta_{b;\,b-1,\ldots,1}$ by

$$\left.\begin{aligned} \zeta^2_{b;\,b-1,\ldots,1} &= q^2_{12\ldots b} - q^2_{12\ldots b-1} \\ \vdots \\ Z^2_n &= r^2 - q^2_{12\ldots n-1} \end{aligned}\right\} \quad \ldots\ldots \quad \textbf{Ib}$$

Of course, we have analogous controlling relations for the corresponding magnitudes $\zeta_{p_b;\,p_1,\ldots,p_{b-1}}$ $(b>2)$.

The magnitudes $\zeta_{b;\,b-1,\ldots,1}$ may just as well be controlled by the equations **57bis**:

$$\left.\begin{aligned} \zeta_{3;21} &= \Big(\sum_{\alpha=1}^{3} \overset{1.2.3}{\Gamma}_{\alpha 3}\,.\,z_\alpha\Big) : \sqrt{\overset{1.2.3}{(\Gamma}\,.\,\overset{1.2.3}{\Gamma_{33})}} \\ \vdots \\ \zeta_{b;\,b-1,\ldots,1} &= \Big(\sum_{\alpha=1}^{b} \overset{1\ldots.b}{\Gamma}_{\alpha b}\,z_\alpha\Big) : \sqrt{\overset{1\ldots.b}{(\Gamma}\,.\,\overset{1\ldots.b}{\Gamma_{bb})}} \\ \vdots \\ Z_n &= \Big(\sum_{\alpha=1}^{n} \Gamma_{\alpha n}\,.\,z_\alpha\Big) : \sqrt{(\Gamma\,.\,\Gamma_{nn})} \end{aligned}\right\} \quad \ldots \quad \textbf{Ibbis}$$

So the supposition $t_a\,\{x_a\}$, with which we started, must be confirmed by the controlling relations **Ia** and **Ibbis**. But it must besides appear that

$$M_{ab} = \left(\operatorname{tg}\omega_{ab} = \frac{\sqrt{1-\gamma^2_{ab}}}{\gamma_{ab}} =\right) \frac{B_{ab}}{A_{ab}} = \text{constant.}$$

By taking for a and b all the combinations of the subscripts $1, 2, \ldots, n$, we obtain from **Ia** $\dfrac{n\,(n-1)}{2}$ controlling equations.

By replacing, in $\zeta_{3;21}\,(=\zeta_{3;12})$ the triplet of subscripts 1, 2, 3 by other triplets, we obtain together $\dfrac{n!}{1!\,2!\,(n-3)!} = \dfrac{n\,(n-1)\,(n-2)}{2!}$ similar magnitudes $\zeta_{p_3;\,p_1\,p_2}$.

Likewise $\zeta_{4;\,321}\,(=\zeta_{4;\,312}=\ldots.=\zeta_{4;\,123})$ furnishes together $\dfrac{n!}{1!\,3!\,(n-4)!} =$ $= \dfrac{n\,(n-1)\,(n-2)\,(n-3)}{3!}$ analogous magnitudes $\zeta_{p_4;\,p_1\,p_2\,p_3}$, and $\zeta_{b;\,b-1,\ldots,1}$ $\dfrac{n!}{1!\,(b-1)!\,(n-b)!} = \dfrac{n\,(n-1)\ldots(n-b+1)}{(b-1)!}$ corresponding magnitudes $\zeta_{p_b;\,p_1\ldots p_{b-1}}$, and finally Z_n n similar magnitudes Z_p. Hence the total number of conditions **Ib** amounts to $\sum_{b=3}^{n} \dfrac{n\,(n-1)\ldots(n-b+1)}{(b-1)!}$, or, putting $b-1=\beta$,

$$\sum_{\beta=2}^{n-1} \frac{n\,(n-1)\ldots(n-\beta)}{\beta!} = n\,.\sum_{\beta=2}^{n-1} \frac{(n-1)!}{\beta!\,(n-1-\beta)!} =$$

$$= n\,\{(1+1)^{n-1} - 1 - (n-1)\} = n\,(2^{n-1}-n).$$

The $\dfrac{n\,(n-1)}{2}$ conditions **Ia** concede to the $n + n\,(n-1) = n^2$ magnitudes $z_a, \zeta_{b;a}$ still $n^2 - \dfrac{n\,(n-1)}{2} = \dfrac{n\,(n+1)}{2}$ degrees of freedom. These

are just necessary for determining 1^0 the $\frac{n(n-1)}{2}$ elements (arcs) ω_{ab}, 2^0 the n coordinates of the image point P representing the set $(x_1, \ldots, x_n)$.

Summarizing the results arrived at above, we may conclude that — in order to confirm the supposition that t_a is a function of only x_a $(a = 1, \ldots, n)$ — the following conditions must be satisfied:

$$z_a^2 + \zeta_{b;a}^2 = z_b^2 + \zeta_{a;b}^2 = q_{ab}^2 \quad ; \quad \left[\frac{n(n-1)}{2} \text{ cond.}\right] \quad . \quad . \quad . \quad \text{Ia}$$

$$\left.\begin{aligned} \zeta_{3;21} &= \left(\sum_{\alpha=1}^{3} {}^{1,2,3}\Gamma_{\alpha 3} \,.\, z_\alpha\right) : \sqrt{{}^{1,2,3}(\Gamma \,.\, {}^{1,2,3}\Gamma_{33})}, \\ &\vdots \\ \zeta_{b;b-1,\ldots,1} &= \left(\sum_{\alpha=1}^{b} {}^{1,\ldots,b}\Gamma_{\alpha b} \,.\, z_\alpha\right) : \sqrt{{}^{1,\ldots,b}\Gamma \,.\, {}^{1,\ldots,b}\Gamma_{bb}}, \\ &\vdots \\ Z_n &= \left(\sum_{\alpha=1}^{n} \Gamma_{\alpha n} \,.\, z_\alpha\right) : \sqrt{(\Gamma \,.\, \Gamma_{nn})}; \end{aligned}\right\} [n(2^{n-1} - n) \text{ cond.}] \,. \quad \text{Ibbis}$$

$$M_{ab} = \left(\text{tg } \omega_{ab} = \frac{\sqrt{1-\gamma_{ab}^2}}{\gamma_{ab}} =\right) \frac{B_{ab}}{A_{ab}} = \text{constant} \left[\frac{n(n-1)}{2} \text{ cond.}\right] \,. \quad \text{II}$$

Then the functions t_a are determined by

$$t_a = (\Gamma_{aa} : \Gamma)^{1/2} \,.\, z_a \qquad (a = 1, \ldots n). \quad . \quad . \quad . \quad . \quad . \quad \text{J}$$

In the particular case that all the functions $z_a \{x_a\}$ $(a = 1, \ldots, n)$ are *linear*, there exists already linear correlation between the original variables $x_1, \ldots, x_n$ themselves. [1])

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[1]) Having to do with only one variable, we may test the linearity of the correlation also by the conditions that the "skewness" S and the "excess" E vanish. The notions "skewness" and "excess" may easily be applied also in the case of n variables.

Then, taking for instance the variable x_a, we operate with the class-*centres* $\xi_a(k_a)\,(k_a = 1, \ldots, \nu_a)$ and with the deviations $u_a(k_a) = \xi_a(k_a) - \bar{\xi}_a$ of those $\xi_a(k_a)$ from their average value $\bar{\xi}_a$. Representing, in general, the average value of a function $f\{u_1, u_2, \ldots, u_n\}$ of the magnitudes $u_1, u_2, \ldots, u_n$ by $\overline{f\{u_1, u_2, \ldots, u_n\}}$, this average has for its theoretical value the "mathematical expectation" $\int f\{u_1, u_2, \ldots, u_n\}\, dW$, its empirical value being

$$\frac{1}{N} \cdot \sum_{i_1=1}^{\nu_1} \sum_{i_2=1}^{\nu_2} \ldots \sum_{i_n=1}^{\nu_n} Y(i_1, i_2, \ldots, i_n) \,.\, f\{u_1(i_1), u_2(i_2), \ldots, u_n(i_n)\}.$$

Denoting moreover the "mean" value of u_a by ε_a, that is to say, putting $\varepsilon_a = \sqrt{\overline{u_a^2}}$ (whence $\overline{u_a u_b} = \gamma_{ab}\, \varepsilon_a\, \varepsilon_b$), then linearity of correlation can theoretically be controlled by the conditions:

$$\overline{u_a^3} = 0\,[n \text{ cond.}],\ \overline{u_a^2 u_b} = 0\,[n(n-1) \text{ cond.}],\ \overline{u_a\, b_b\, u_c} = 0\left[\frac{n(n-1)(n-2)}{6} \text{ cond.}\right];$$

If the conditions **Ia, Ib(bis)** are satisfied, but the conditions **II** are *not*, then we *introduce constant* arcs $\overline{\omega}_{ab}$, hence *constant* (total) coefficients of correlation $\overline{\gamma}_{ab} = \cos \overline{\omega}_{ab}$. The magnitudes $\overline{\omega}_{ab}$ are preferably obtained by averaging M_{ab} (see S. C. I. *b*, Dutch text p. 976, English text p. 929) and by computing the arcs $\overline{\omega}_{ab} = \text{arc tg } \overline{M}_{ab}$ from those averages $\overline{M}_{ab}$. According to **32**, we put

$$\left.\begin{aligned}
z_1 &= (\overline{\Gamma} : \overline{\Gamma_{11}})^{1/2} \,.\, T_1 \\
\zeta_{2;1} &= (\overline{\Gamma} : \overline{\Gamma}^{1,2})^{1/2} \,.\, (-\overline{\gamma}_{21} \,.\, T_1 : \surd \overline{\Gamma}_{11} + T_2 : \surd \overline{\Gamma}_{22}) \\
&\vdots \\
\zeta_{b;\,b-1,\ldots,1} &= \{\overline{\Gamma} : (\overline{\Gamma^{1,\ldots,b}_{1,\ldots,b}} \,.\, \overline{\Gamma_{bb}})\}^{1/2} \,.\, \sum_{\alpha=1}^{b} \{\overline{\Gamma_{\alpha b}} \,.\, T_\alpha : \surd \overline{\Gamma}_{\alpha\alpha}\} \\
&\vdots \\
Z_n &= (1 : \overline{\Gamma}_{nn})^{1/2} \,.\, \sum_{\alpha=1}^{n} \{\overline{\Gamma}_{\alpha n} \,.\, T_\alpha : \surd \overline{\Gamma}_{\alpha\alpha}\}
\end{aligned}\right\} \quad . \quad . \quad 74$$

By equalizing these expressions for $z_1, \ldots, Z_n$ to the right sides of **57bis**, we may express $T_1, T_2, \ldots T_n$ in the magnitudes $z_1, z_2, \ldots, z_n$ and γ_{ab} $(a, b = 1, \ldots, n)$.

$\overline{u_a^4} = 3\,\varepsilon_a^4$ [n cond.], $\overline{u_a^3 u_b} = 3\,\gamma_{ab}\,\varepsilon_a^3\,\varepsilon_b$ [$n\,(n-1)$ cond.],

$$\overline{u_a^2 u_b^2} = (1 + 2\,\gamma_{ab}^2)\,\varepsilon_a^2\,\varepsilon_b^2 \left[\frac{n\,(n-1)}{2} \text{ cond.}\right],$$

$$\overline{u_a^2 u_b u_c} = (\gamma_{bc} + 2\,\gamma_{ab}\,\gamma_{ac})\,\varepsilon_a^2\,\varepsilon_b\,\varepsilon_c \left[\frac{n\,(n-1)\,(n-2)}{2} \text{ cond.}\right],$$

$$\overline{u_a\,u_b\,u_c\,u_d} = (\gamma_{ab}\,\gamma_{cd} + \gamma_{ac}\,\gamma_{bd} + \gamma_{ad}\,\gamma_{bc})\,\varepsilon_a\,\varepsilon_b\,\varepsilon_c\,\varepsilon_d \left[\frac{n(n-1)(n-2)(n-3)}{24} \text{ cond.}\right].$$

Hence the following conditions must be satisfied:

$$S_{aaa} \equiv \frac{\overline{u_a^3}}{\varepsilon_a^3} = 0, \quad S_{aab} \equiv \frac{\overline{u_a^2 u_b}}{\varepsilon_a^2\,\varepsilon_b} = 0, \quad S_{abc} \equiv \frac{\overline{u_a\,u_b\,u_c}}{\varepsilon_a\,\varepsilon_b\,\varepsilon_c} = 0;$$

$$E_{aaaa} = \frac{\overline{u_a^4}}{\varepsilon_a^4} - 3 = 0, \quad E_{aaab} \equiv \frac{\overline{u_a^3 u_b}}{\varepsilon_a^3\,\varepsilon_b} - 3\,\gamma_{ab} = 0,$$

$$E_{aabb} \equiv \frac{\overline{u_a^2 u_b^2}}{\varepsilon_a^2\,\varepsilon_b^2} - (1 + 2\,\gamma_{ab}^2) = 0, \qquad E_{aabc} \equiv \frac{\overline{u_a^2\,u_b\,u_c}}{\varepsilon_a^2\,\varepsilon_b\,\varepsilon_c} - (\gamma_{bc} + 2\,\gamma_{ab}\,\gamma_{ac}) = 0,$$

$$E_{abcd} \equiv \frac{\overline{u_a\,u_b\,u_c\,u_d}}{\varepsilon_a\,\varepsilon_b\,\varepsilon_c\,\varepsilon_d} - (\gamma_{ab}\,\gamma_{cd} + \gamma_{ac}\,\gamma_{bd} + \gamma_{ad}\,\gamma_{bc}) = 0.$$

The total number of conditions $S = 0$ is equal to the number of combinations with repetition of n elements, taken 3 at a time, thus $\frac{n(n+1)(n+2)}{6}$. The total number of conditions $E = 0$ is similarly found to be $\frac{n(n+1)(n+2)(n+3)}{24}$.

The formulae **74** may be simplified by putting

$$T_a = (\overline{\Gamma}_{aa} : \overline{\Gamma})^{1/2} . U_\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 75$$

Then between the magnitudes U_a there exists just as well linear correlation, the (total) coefficients of correlation γ_{ab} being the same. However, the magnitudes U_a are no longer unimodular, the modulus of U_a being

$$h_a = (\overline{\Gamma}_{aa} : \overline{\Gamma})^{1/2}.$$

The equations **74** now take the following form:

$$\left.\begin{aligned} z_1 &= U_1 \\ \zeta_{2;1} &= (-\overline{\gamma}_{21} U_1 + U_2) : \sqrt{\overline{\Gamma}^{\overline{1;2}}} \\ &\vdots \\ \zeta_{b;b-1,\ldots,1} &= (\sum_{\alpha=1}^{b} \overline{\Gamma}^{\overline{1,\ldots,b}}_{\alpha b} . U_\alpha) : \sqrt{(\overline{\Gamma}^{\overline{1,\ldots,b}} . \overline{\Gamma}^{\overline{1,\ldots,b}}_{bb})} \\ &\vdots \\ Z_n &= (\sum_{\alpha=1}^{n} \overline{\Gamma}_{\alpha n} . U_\alpha) : \sqrt{(\overline{\Gamma} . \overline{\Gamma}_{nn})} \end{aligned}\right\} \quad . \quad . \quad . \quad 74^{bis}$$

Equalizing these expressions for $z_1, \ldots, Z_n$ to the expressions **57bis**, we obtain:

$$\left.\begin{aligned} U_1 &= z_1 \\ (-\overline{\gamma}_{21} U_1 + U_2) : \sqrt{\overline{\Gamma}^{\overline{1;2}}} &= (-\gamma_{21} z_1 + z_2) : \sqrt{\Gamma^{1;2}} \\ &\vdots \\ (\sum_{\alpha=1}^{b} \overline{\Gamma}^{\overline{1,\ldots,b}}_{\alpha b} . U_\alpha) : \sqrt{(\overline{\Gamma}^{\overline{1,\ldots,b}} . \overline{\Gamma}^{\overline{1,\ldots,b}}_{bb})} &= (\sum_{\alpha=1}^{b} \Gamma^{1,\ldots,b}_{\alpha b} . z_\alpha) : \sqrt{(\Gamma^{1,\ldots,b} . \Gamma^{1,\ldots,b}_{bb})} \\ &\vdots \\ (\sum_{\alpha=1}^{n} \overline{\Gamma}_{\alpha n} . U_\alpha) : \sqrt{(\overline{\Gamma} . \overline{\Gamma}_{nn})} &= (\sum_{\alpha=1}^{n} \Gamma_{\alpha n} . z_\alpha) : \sqrt{(\Gamma . \Gamma_{nn})} \end{aligned}\right\} \quad . \quad . \quad \mathbf{K}$$

From these equations we can solve $U_1, \ldots, U_n$ in terms of $z_1, \ldots, z_n$ and γ_{ab} $(a, b = 1, \ldots, n)$. In virtue of **E**, the magnitude $\gamma_{\alpha\beta}$ is built up of $z_\alpha, z_\beta, \zeta_{\beta;\alpha}\, \zeta_{\alpha;\beta}$. Now z_α (see **B**$^{(1)}$) is a function of only x_α, z_β of only x_β, $\zeta_{\beta;\alpha}$ and $\zeta_{\alpha;\beta}$ (see **B**$^{(2;1)}$) being however functions of both variables x_α and x_β. Hence $\gamma_{\alpha\beta}$ is a function of the two variables x_α and x_β. From this ensues, that the determinant

$$\Gamma^{1,\ldots,b} = \begin{vmatrix} \gamma_{11}, & \gamma_{12}, \ldots, & \gamma_{1b} \\ \gamma_{21}, & \gamma_{22}, \ldots, & \gamma_{2b} \\ \vdots & \vdots & \vdots \\ \gamma_{b1}, & \gamma_{b2}, \ldots, & \gamma_{bb} \end{vmatrix}$$

is a function of the b variables $x_1, x_2, \ldots x_b$, but independent of $x_{b+1}, \ldots x_n$.

From the structure of the equations **K** it appears, that U_1 is a function of only x_1, U_2 of x_1 and $x_2, \ldots, U_b$ of $x_1, \ldots, x_b, \ldots, U_n$ of *all* the variables $x_1, \ldots, x_n$.

Thus we may formulate the result as follows:

If the conditions **Ia**, **Ib**$^{(bis)}$ are satisfied, but the conditions **II** are *not*,

then it is possible to establish linear correlation between the variables $U_a\,(a=1,\ldots,n)$ built up in such a way that U_b is a function of the b variables $x_1,\ldots,x_b$, or, in other words, that we have for $U_b\{x_1, x_2,\ldots,x_b\}$:

$$\frac{\partial U_b}{\partial x_a}=0, \quad \text{for} \quad a>b.$$

Here, too, we might consider the particular cases that some of the conditions **II** are fulfilled, the others *not*. These cases can be treated according to the model explained for three variables. The most important property might be, that, in the case $\gamma_{12}=\text{const.}$, U_2 becomes a function of only x_2.

If **Ia** is still fulfilled, but **Ib(bis)** is not, then we introduce "adjusted" values $\zeta'_{b;\,b-1,\ldots,1}\,(b>2)$, which do equal the values computed from the expressions **Ib(bis)** in z_α, $\gamma_{\alpha\beta}\,(\alpha,\beta=1,\ldots,b)$. Thus they are *defined* by

$$\zeta'_{b;\,b-1,\ldots,1}=\left(\sum_{\alpha=1}^{b}{}^{1,\ldots,b}\,\Gamma_{\alpha b}\,.\,z_\alpha\right):\sqrt{\left(\Gamma^{1,\ldots,b}_{1,\ldots,b}\,.\,\Gamma_{bb}\right)} \quad . \quad . \quad . \quad . \quad \mathbf{77}$$

So we obtain together $n\,(2^{n-1}-n)$ magnitudes. By replacing the values $\zeta_{b;\,b-1,\ldots,1}$ — empirically found — by the adjusted values **77**, we satisfy artificially **Ib(bis)**. Then the formulae **K** need no alteration. Now the magnitudes z_α, $\zeta_{\alpha\beta;\,\alpha}$, $\zeta'_{b;\,b-1,\ldots,1}$ etc. do not give back the original frequency distribution, but an "adjusted" frequency distribution. As a rule the discrepancy will be of little importance. As the magnitudes $\zeta_{b;\,b-1,\ldots,1}$, with increasing b, become still less certain (more interpolations, smaller frequencies), we shall seldom desire, intentionally, to keep their empirical values. If we, nevertheless, wish to retain one or more of the empirical values, we must alter one or more of the elements ω_{ab}, whence the formulae **K**, in consequence, undergo alteration.

If even **Ia** is not satisfied, so that $q_{b;a}\neq q_{a;b}$, then we operate preferably with the magnitudes z_a and with the magnitudes γ_{ab} (resp. ω_{ab}) computed from **E**. In this case, however, we cannot retain the empirical values $\zeta_{b;a}$, $\zeta_{a;b}$. In their place we must use the "adjusted" values:

$$\left.\begin{aligned}\zeta'_{b;a}&=\frac{z_b-\cos\omega_{ab}\,.\,z_a}{\sin\omega_{ab}}=\frac{-\gamma_{ab}\,z_a+z_b}{\sqrt{1-\gamma^2_{ab}}},\\ \zeta'_{a;b}&=\frac{z_a-\cos\omega_{ab}\,.\,z_b}{\sin\omega_{ab}}=\frac{z_a-\gamma_{ab}\,z_b}{\sqrt{1-\gamma^2_{ab}}},\end{aligned}\right\} \quad . \quad . \quad . \quad \mathbf{78}$$

which now, in virtue of their definition, satisfy the second row of the equations **57(bis)**. They also furnish, together with z_a, z_b, automatically the same $\omega_{ab}\,(\gamma_{ab})$ as the empirical values.

The magnitude q_{ab}^2 must now be replaced by

$$q_{ab}'^2 = \frac{z_a^2 - 2\cos\omega_{ab} \cdot z_a z_b + z_b^2}{\sin^2\omega_{ab}} \quad \ldots\ldots\ 79$$

This expression is obtained by generalizing the formula, which derives from 70(1) by putting $b = 2$.

Since, also in this case, the magnitudes z_a, ω_{ab} (γ_{ab}) are computed from the same data as before, we may still keep the equations K unaltered. These must only then undergo alteration, if we desire, intentionally, to retain the empirical values of a certain set of variables, e.g. of z_1, $\zeta_{2;1}$, $\zeta_{3;21}, \ldots, \zeta_{b;\,b-1\ldots,1}, \ldots, Z_n$.

Conclusion:

Any frequency distribution of n variables $x_1, x_2, \ldots, x_n$ can *always* be interpreted by a *linear* correlation between n variables U_a $(a = 1, 2, \ldots, n)$, U_b being a function of the b variables $x_1, x_2, \ldots, x_b$, and independent of the others $(x_{b+1}, \ldots, x_n)$.

If, in particular, we find, for each set x_a (k_a) and x_b (k_b) $(k_a = 1, \ldots, \nu_a,\ k_b = 1, \ldots, \nu_b)$, one and the same value of the magnitude $M_{ab} = \frac{z_a \zeta_{a;b} + z_b \zeta_{b;a}}{z_a z_b - \zeta_{b;a}\zeta_{a;b}}$ (corresponding to that pair of variables), then we can make U_a — hence also the corresponding unimodular variable T_a — a pure function of x_a. Then the (total) coefficients of correlation are $\gamma_{ab} = \cos\omega_{ab}$, being $M_{ab} = \operatorname{tg}\omega_{ab}\left(\frac{\cos\omega_{ab}}{A_{ab}} > 0\right)$.

Anatomy. — *A note on the optic tract in teleosts.* By J. JANSEN. (Fellow of the Rockefeller Foundation.) (From the Central Dutch Institute for Brain-Research, Amsterdam.) (Communicated by Prof. C. U. ARIËNS KAPPERS.)

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Owing to the intimate relation of the optic tract to the different components of the postoptic commissures, the determination of the different components of the former and their course offers many difficulties. A brief review of the literature on the subject will bring out the controversies which are pending, and indicate the problems which still remain unsettled.

Our survey may commence with BELLONCI's (1888) classical comparative study of the relations of the optic tract in vertebrates, which also deals with the optic system of teleosts. BELLONCI distinguishes three divisions of the optic tract. (1) The main lateral division runs from the chiasma, where a complete crossing of the fibres takes place, dorsocaudad, sweeping along the lateral aspect of the thalamus, in order to reach the tectum opticum, where the fibres terminate in the superficial fibrous layer. This division of the optic tract is subdivided by the lateral geniculate body into a dorsal and ventral fascicle (here called fasciculus dorsalis et lateralis tractus optici). The fibres discharge collaterals into the geniculate body, but they themselves all seem to end in the tectum opticum. (2) A small portion of the optic fibres caudally to the chiasma becomes detached from the main lateral division. This component, evidently the fasciculus medialis tr. optici of subsequent authors, accompanies the commissura minor (HERRICK) first in a dorsal, then in a lateral direction towards the posteroventral aspect of the geniculate body, where the bundle rejoins the fasciculus lateralis of the main lateral division. Finally, BELLONCI describes as a component of the optic tract (3) a bundle of fibres which accompany the fasciculus dorsalis, lying immediately medial to it. This component likewise enters the tectum opticum, and evidently corresponds to the fibrae tectales n. optici of more recent investigators. BELLONCI holds that all optic fibres terminate in the tectum opticum, but some of them are related by collaterals to the thalamic nuclei, especially to the geniculate body.

The next investigator to deal extensively with the optic tract of bony fishes is KRAUSE (1898). Employing the Marchi method after enucleation of the eye he could confirm BELLONCI's description of the main division (fasciculus dorsalis et lateralis) of the optic tract. As regards the third division of the optic tract, the "fibrae tectales n. optici", they appeared not to be affected by the enucleation. Marchi preparations showed no

degeneration within this bundle. In a Weigert series of a specimen with a degenerated optic tract, this bundle alone stained normally. Finally, in a series of a fish which was blind on one eye the fibrae tectales n. optici were still present. From these facts KRAUSE concludes that the optic tract contains a fibre-system which does not degenerate after removal of the eye. He considers it a centrifugal system connecting the tectum opticum with the eye. In order to prove this conclusion KRAUSE performed experiments by making injuries to the tectum. In all but one of these experiments the fishes died too soon after the operation. The one experiment which was fairly successful tends to show that the fibres in question form a tectofugal system ; but KRAUSE fails to prove that they constitute a part of the optic tract. Neither in the text nor in the figures is there any indication that he ever tried to trace the fibres beyond the chiasma into the optic nerve. KRAUSE completely disregards the possibility that we may here be dealing with a commissural system belonging to the postoptic commissure complex.

FRANZ (1912) in a paper on the di- and mesencephalon of teleosts describes a new fibre-bundle, which he calls the tr. isthmo-opticus, and which he considers a part of the optic system. Anteriorly the tr. isthmo-opticus accompanies the optic tract. Coursing spinalwards it joins the commissura transversa, curves around the torus semicircularis, and enters into relation with the ganglion isthmi. On the basis of morphological and experimental evidence FRANZ arrives at the conclusion that the tract in question arises in the ganglion isthmi, and runs centrifugally through the optic nerve into the eye.

The optic system of bony fishes is next taken up for discussion by WALLENBERG (1913), who studied a Weigert series of a trout embryo, the left eye of which was gone. His findings concerning the fasciculi dorsalis et lateralis of the optic tract corroborate BELLONCI's observations. In addition to these components WALLENBERG describes a bundle of optic fibres which run deeper in the brain tissue caudad and dorsad towards a nucleus ("ganglion opticum mediale") which lies lateral to the central gray of the third ventricle. WALLENBERG confirms this observation experimentally on telescope fishes, a variety of the goldfish with hypertrophied eyes. (Marchi-method) remarking (p. 274) : "Dabei konnte ich ebenfalls eine intrathalamische Endigung medialster Fasern des Tractus opticus, allerdings in einem dorsaler gelegenen Ganglion, lateral vom Kern des Fasciculus longitudinalis dorsalis, nachweisen". — WALLENBERG also observed the tr. isthmo-opticus of FRANZ, which he, without committing himself, is inclined to regard as a commissural system.

HOLMGREN (1920) in his comprehensive study of the telencephalon and diencephalon of teleosts describes the following new components of the optic nerve for Osmerus eperlanus : (a) *Tr. olfactorius lateralis optici*. This bundle, which forms a part of the tr. olfactorius lateralis, HOLMGREN was able to trace spinalwards into the optic tract, and thence for some distance peripherally into the optic nerve. In Callionymus lyra he observed a similar

bundle within the medial olfactory tract. (b) *Tr. recesso-opticus.* Is derived from the nucleus preopticus pars recessi. "Die Neuriten treten in den Opticus mit zentrifugalem Verlauf ein. Wohin sie im Opticus ziehen, konnte nicht festgestellt werden." (op. cit., p. 222). (c) *Tr. preoptico-opticus posterior.* In the chiasma region a small fibre-bundle becomes detached dorsally from the main mass of optic fibres. This bundle HOLMGREN traced caudalwards into the posterior part of the nucleus parvocellularis. In Cajal preparations he could demonstrate that "einzelne Fasern des Tractus in die birnförmigen Zellkörper von hier gelegenen Ganglienzellen eintreten." (op. cit., p. 224).

The olfacto-optic connections mentioned above were reinvestigated by KUDO (1923). He could confirm HOLMGREN's observations on Callionymus. Furthermore, he observed similar fibres within the medial olfactory tract in several other teleosts (tr. olfactorio-opticus). KUDO was unable, however, to determine whether the fibres take a peripheral course into the optic nerve in the chiasma region, or proceed spinalwards within the brain.

The presence of the tr. isthmo-opticus (FRANZ) is confirmed by KUDO in the paper referred to above; but this investigator arrives at an entirely different conclusion regarding the rostral course of the tract. According to KUDO the tract anteriorly does not accompany the optic fibres towards the chiasma, but turns dorsad to terminate in the rostral part of the tectum. He names the bundle tr. isthmo-tectalis [1]).

From this short survey of the literature it is seen that the following components have been described in the optic system of teleosts:

(1). *The main, lateral (marginal) division,* comprising a dorsal and ventral (here called lateral) fascicle (BELLONCI, KRAUSE, WALLENBERG), the fibres of which all terminate in the tectum opticum.

(2). *Fasciculus medialis tr. optici* (BELLONCI), an aberrant bundle of optic fibres which rejoin the main division laterally.

(3). *Fibrae tectales n. optici* (BELLONCI, KRAUSE). BELLONCI regards this as a tectopetal connection (afferent optic fibres), whereas KRAUSE arrives at the conclusion that the fibres originate in the tectum, and run centrifugally into the optic nerve (efferent optic fibres).

(4). A component (WALLENBERG) which ends in the thalamus ("ganglion opticum mediale") (trout) or in the midbrain in the vicinity of the nucleus of the fasciculus longitudinalis medialis (goldfish).

(5). *Tractus isthmo-opticus* (FRANZ) = *Tr. isthmo-tectalis* (KUDO). By FRANZ assumed to represent a centrifugal connection between the tectum and the retina; by KUDO, on the other hand, considered an isthmo-tectal connection.

(6). *Tractus olfactorius lateralis optici* (HOLMGREN), fibres which originate in the olfactory bulb, and run centrifugally into the optic nerve.

[1]) In some species of teleosts KUDO found that the fibres of this tract end in the nucleus pretectalis. In this case he, in addition to a tr. isthmo-tectalis, speaks of a tr. isthmo-pretectalis or pretecto-isthmicus.

(7). *Tractus recesso-opticus* (HOLMGREN).

(8.) *Tractus preoptico-opticus posterior* (HOLMGREN). The two latter tracts from the preoptic nucleus course peripherally into the optic nerve.

As may be seen from this review there still exist many discrepancies concerning the optic tract of bony fishes. Having the opportunity, through the courtesy of Professor KAPPERS, who placed the large collection of the Central Dutch Institute for Brain-Research at my disposal, to examine the teleostean brain, I undertook to restudy their optic system in order to determine: (1) which are the components of the optic tract in teleosts, (2) what are their central connections, (3) are the so-called fibrae tectales n. optici, the tr. isthmo-opticus and the tr. olfactorius lateralis optici components of the optic nerve, (4) if so, are they retino-petal or retino-fugal in nature, (5) to what extent are the components of the optic tract of teleosts comparable with the components of the optic tract of higher vertebrates?

In the collection of the Institute there is a Weigert—Pal series of the brain of a teleost, Idus idus, the left eye of which was gone by disease [1]). This series proved very suitable for the purpose of the present investigation, the results of which are mainly derived from a study of this specimen.

A description of the accompanying figures, which were drawn with the Edinger projection apparatus, and reproduce selected sections of the blind Idus idus, will bring out the facts which are of interest in this connection.

ABBREVIATIONS FOR ALL FIGURES.

br. t. dors. = brachium tecti dorsale.
com. hab. = commissura habenularis.
com. hor. = commissura horizontalis.
com. intergen. = commissura intergeniculata.
com. post. = commissura posterior.
com. t. = commissura tecti.
com. tr. = commissura transversa.
corp. gen. lat. = corpus geniculatum laterale.
f. ans. = fibrae ansulatae.
f. dors. tr. op. = fasciculus dorsalis tr. optici.
f. dors. med. tr. op. = fasciculus dorsomedialis tr. optici.
f. lat. tel. = fasciculus lateralis telencephali.
f. lat. tr. op. = fasciculus lateralis tr. optici.
f. med. tel. = fasciculus medialis telencephali.
f. med. tr. op. = fasciculus medialis tr. optici.
f. retr. = fasciculus retroflexus.
f. t. op. = fibrae tectales nervi optici.
hab. = habenula.
hem. = hemisphere.
nuc. po. = nucleus preopticus.
nuc. rot. = nucleus rotundus (corpus glomerosum, FRANZ).
str. med = stria medullaris.
tect. = tectum opticum.
tr. gen. t. = tractus geniculo-tectalis.
tr. i. t. = tractus isthmo-tectalis (KUDO).
tr. op. = tractus opticus.
tr. po. gen. hyp. = tractus preoptico-geniculo-hypothalamicus.
tr. po. op. = tractus preoptico-opticus.
tr. t. b. = tractus tecto-bulbaris.
tr. t. cer. = tractus tecto-cerebellaris.

Fig. 1 is drawn from a section immediately spinalward of the chiasma The section is somewhat oblique, the left side representing a more caudal

[1]) There is no trace of the left optic nerve in the series, of the right optic the central stump is preserved.

level, which explains the considerable difference in the size of the hemispheres. On the left side the picture is dominated by the large left

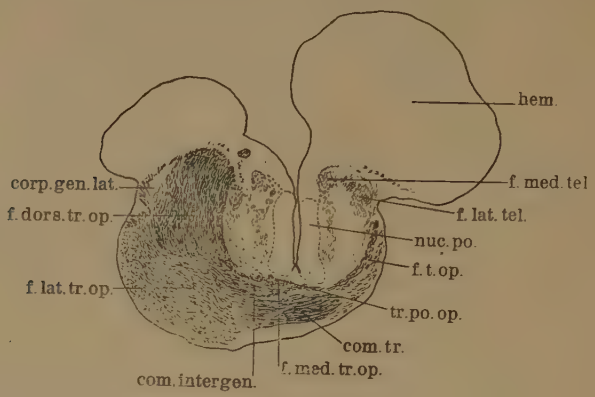

Fig. 1. Idus idus, blind on the left eye. Cross section through the chiasma region × $13^1/_2$.

optic tract. Dorsolaterally the rostral extremity of the geniculate body is transected. On the right side the position of the optic tract is occupied by two bundles running parallel in a dorsolateral direction. The medial bundle consists of heavier, more densely packed fibres. These are the fibrae tectales nervi optici of KRAUSE. On the left side they cannot, at this level, be differentiated from the optic tract. The lateral, superficial bundle on the right side consists of paler, more loosely arranged fibres, the significance of which is unknown. Centrally the large preoptic nucleus is transected. Immediately ventral to the left preoptic nucleus may be seen a small round bundle of fibres which from the chiasma can be traced in a dorsomedial direction. The bundle disappears in the region between the dorsal border of the postoptic commissures and the bottom of the ventricle, close to the latter, at the level of the crossing of the commissura horizontalis. The fact that the bundle is lacking on the right side is in favour of its being formed by fibres belonging to the optic nerve. It should be stated, however, that the fibres which constitute this small bundle are of considerably finer caliber than the remainder of the optic fibres. In a normal specimen of Idus idus the bundle was present bilaterally. I take this to be the tr. preoptico-opticus posterior of HOLMGREN. Ventral to the tract under consideration is found another, considerably larger bundle, which is also unilateral in the blind specimen. This is the fasciculus medialis tr. optici.

Fig. 2 is drawn from a section which cuts through the thalamus, and on the left side also transects the anterior pole of the tectum opticum. The left optic tract is divided by the geniculate body and the nucleus rotundus (corpus glomerosum, FRANZ) into a fasciculus dorsalis and a fasciculus

lateralis. The fibres of the former are spreading fan-like over the anterior pole of the tectum. The fibrae tectales n. optici are also now recognizable

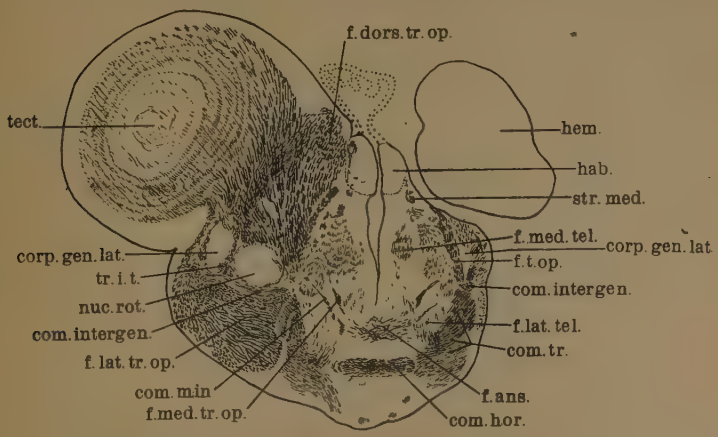

Fig. 2. Idus idus, blind on the left eye. Cross section through the diencephalon, on the left side also cutting the anterior pole of the tectum mesencephali × 13 1/2.

on the left side as a well defined bundle medial to the fasciculus dorsalis. I wish to emphasize that this fibre-system is approximately of equal size on both sides. The tr. preoptico-opticus has already vanished. The fasciculus medialis tr. optici is seen coursing dorsolaterad, accompanying the commissura minor (HERRICK). In the ventral part of the section the crossing of the commissura horizontalis (FRITSCH) is a conspicuous feature. The geniculate body is transected on both sides. There is a pronounced atrophy of the right nucleus.

Fig. 3. Section immediately spinalward of the habenula. The tectum opticum is cut on both sides, and, although the difference in size of the two tecta is partly due to the obliquity of the section, it is evident that there is a considerable atrophy of the right tectum opticum. On the left side the optic fibres are spreading in the superficial layer of the tectum, whereas the so-called fibrae tectales n. optici may be seen entering the deep fibre-layer. As they radiate into the tectum, the fibrae tectales n. optici leave behind a round bundle of fibres, which in fig. 3 is located ventral to the fasciculus dorsalis of the optic tract, lateral to a component of the stria medullaris. This bundle, which is present on the side of the normal optic tract only, I have called the *fasciculus dorsomedialis tr. optici*. Ventral to the geniculate body is found on both sides a small, but distinct bundle of fibres. This is the tr. isthmo-tectalis of KUDO (tr. isthmo-opticus of FRANZ). When traced rostrad, the bundle is seen to bend dorsad, and to enter the anterior pole of the tectum. This is particularly easily observed on the right

side, where no optic fibres obscure the picture. The atrophy of the right geniculate body is very evident.

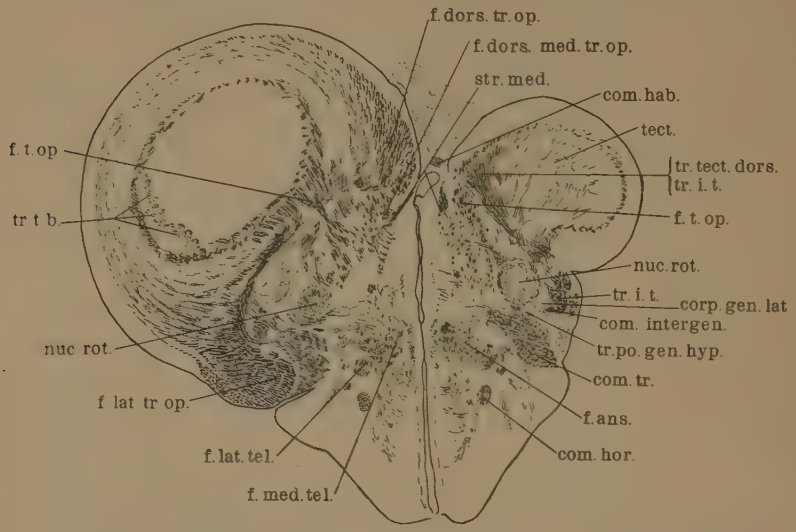

Fig. 3. Idus idus, blind on the left eye. Cross section through the posterior part of the diencephalon, dorsolaterally transecting the tectum mesencephali × 13½.

Fig. 4. From a section through the anterior border of the posterior commissure. To the left of the midline, dorso-lateral to the fibres of the

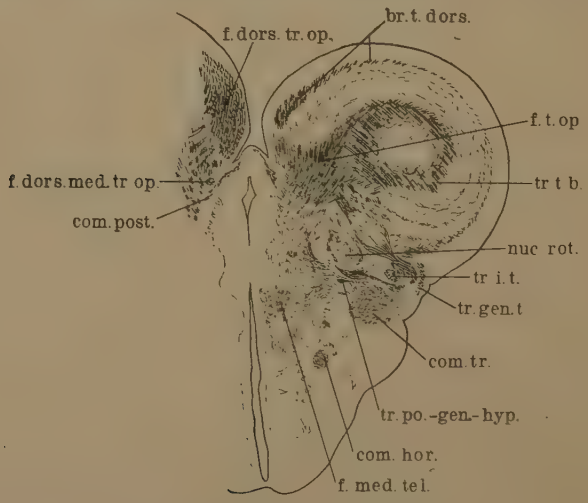

Fig. 4. Idus idus, blind on the left eye. Cross section at the rostral border of the posterior commissure × 13½.

posterior commissure, the fasciculus dorsomedialis forms a distinct bundle. No such bundle is present on the right side. The right tectum opticum is atrophic, the atrophy being particularly pronounced in the superficial layer. The fibrae tectales n. optici are seen to enter the deep layer of the tectum, where they terminate. It should be noticed that this system is not appreciably influenced by the absence of the optic tract on the same side. The tr. isthmo-tectalis forms a conspicuous round bundle ventrolaterally at the basis of the tectum opticum.

Fig. 5. Section through the posterior commissure. The atrophy of the tectum opticum on the right side is evident. The dorsal and lateral fascicles

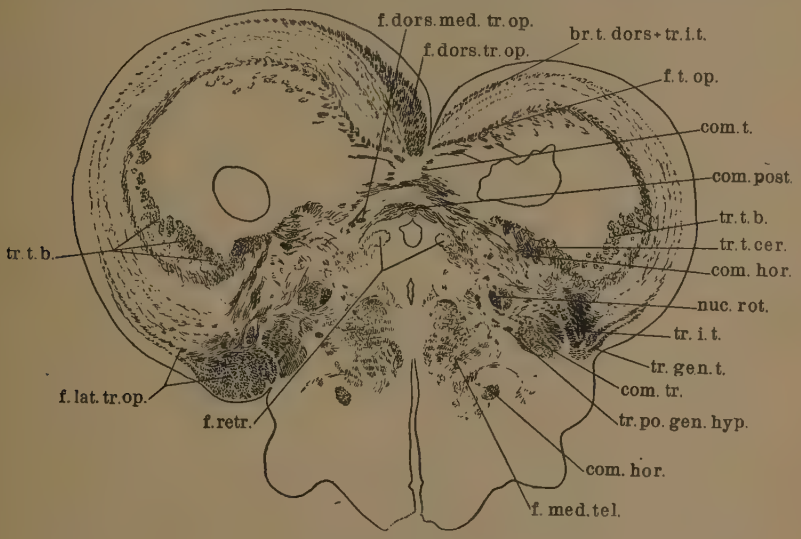

Fig. 5. Idus idus, blind on the left eye. Cross section at the level of the posterior commissure × 13½.

of the normal, left optic tract are spreading over the tectum opticum. The corresponding tectal layer on the right side is represented by a thin sheet of fibres which are derived from the brachium tecti. The fasciculus dorsomedialis tr. optici is found in the left half of the section, lateral to the fibres of the posterior commissure. As in the two preceding figures there is no trace of the bundle on the right side. The tr. isthmo-tectalis is present on both sides, occupying a position lateral to the fibres of the commissura transversa.

In the subsequent sections spinalward the fasciculus dorsalis and lateralis of the optic tract are seen to terminate in their entirety in the optic tectum. The fasciculus dorsomedialis disappears a few sections spinalward of the level of fig. 5, ending dorsolaterally in the tegmentum mesencephali in the nucleus and amongst the bundles of the posterior commissure.

Regarding the tr. olfactorius lateralis opticus I can pronounce no definite opinion. I was unable to demonstrate the tract in my Weigert material of Idus, a fact which, of course, does not exclude the possibility of its existence. Nor could the tr. olfactorio-opticus of KUDO be found in this fish. But in Ammodytes tobianus it was easily recognized. I can present no definite information regarding the destination of these fibres, but agree with KUDO, however, that they seem to proceed in a caudal direction without entering the optic nerve. I think, therefore, that his term tr. olfactorio-opticus is not felicitous, since it tends to create the conception that the tract constitutes a component of the optic nerve.

What conclusions may be drawn from this examination of Idus idus? In a specimen which was blind on one eye, with complete degeneration of the optic nerve, it is seen that the following fascicles described as components of the optic tract were present on one side (contralateral to the normal eye) only: a) *Fasciculus dorsalis tr. optici*; b) *Fasciculus lateralis tr. optici*; c) *Fasciculus medialis tr. optici*; d) *Fasciculus dorsomedialis tr. optici*; e) *Tr. preoptico-opticus*. Two other tracts, likewise sometimes considered components of the optic nerve, were bilaterally represented, viz., a) *fibrae tectales n. optici* and b) *tr. isthmo-tectalis (tr. isthmo-opticus* (FRANZ)).

It is perhaps advisable first to consider the question whether the latter two tracts at all constitute parts of the optic tract. Beginning with the fibrae tectales n. optici, three possibilities present themselves. The first, viz., that the tract is formed by centripetal (afferent) optic fibres is ruled out at once, the tract being bilaterally present, although there is only one optic nerve in the specimen investigated. (From BELLONCI, CAJAL (1899) and others we know that there is a complete crossing of the optic nerves in the chiasma of fishes; hence the tract in question cannot represent an uncrossed optic component). The second possible explanation is advanced by KRAUSE, who considers the fibrae tectales a centrifugal tract composed of fibres which originate in the tectum opticum, and terminate in the eye. This, of course, is conceivable; but it is extremely difficult to bring this conception into accordance with the fact revealed by the examination of the one-eyed specimen of Idus, viz., that there is no appreciable difference in the development of the tracts on both sides, while we should expect a distinct atrophy due to inactivity on the right side where the fibrae tectales n. optici, if constituting a component of the optic nerve, are deprived of their field of termination.

The complete lack of such evidence strongly contradicts KRAUSE's conception, and renders the third possibility which presents itself highly probable, viz., that we are dealing with a commissural connection, a component of the postoptic commissure complex.

As regards the tr. isthmo-opticus of FRANZ, KUDO claims that this tract anteriorly does not join the optic fibres, but bends dorsad and terminates

in the tectum opticum (tr. isthmo-tectalis, KUDO). From my description of Idus it is seen that I agree with KUDO on this point.

My conclusion, then, is that neither the fibrae tectales n. optici, nor the tr. isthmo-tectalis constitute parts of the primary optic system.

The next question which arises is whether the fascicles which were found to be unilaterally present in the one-eyed Idus, are all components of the optic tract. As regards the fasciculus dorsalis and lateralis there can be no doubt in view of the mass of morphological and experimental evidence which has been advanced by previous investigators that these fascicles largely, if not entirely, are formed by optic fibres originating in the retina. The same holds true of the fasciculus medialis tr. optici which forms an aberrant bundle of the fasciculus lateralis. As to the tr. preoptico-opticus and fasciculus dorsomedialis (here described for the first time) my interpreting them as components of the optic tract rests upon the observation that these two bundles are present in the one-eyed specimen of Idus on the side of the normal optic tract only, whereas they are found bilaterally in a normal specimen. My conclusion has not been experimentally confirmed, but it does not seem unlikely that the bundle which WALLENBERG in his Marchi experiments on goldfishes traced to the region lateral to the nucleus of the f.l.m. is myf. dorsomedialis tr. optici. It should be mentioned, furthermore, that LUBSEN (see KAPPERS (1921, p. 1293)), in his very interesting, but unfortunately unpublished investigation of the optic system of Leuciscus rutilus, after laesions in the posterior part of the retina found degenerations (Marchi method) in the dorsal part of the thalamus. These findings are in good accordance with my observations concerning the fasciculus dorsomedialis tr. optici [1]).

If HOLMGREN's tr. preoptico-opticus posterior be compared with my tr. preoptico-opticus little doubt is left that these two tracts are identical. Before I knew HOLMGREN's observations I had described this tract as the fasciculus preopticus tr. optici. The fact that the fibres were considerably finer than the remainder of the optic fibres, made me wonder whether they differed from the retinal fibres in nature. When it appeared that HOLMGREN had observed the tract before, and demonstrated that the fibres took their origin, at least partly, from cells in the preoptic nucleus, I adopted his terminology, leaving out "posterior" as I have observed one optic bundle from this nucleus only.

Accepting these conclusions we find that the optic tract in teleosts (Idus idus), as far as my observations go, consists of the following components: 1) *Fasciculus dorsalis.* 2) *Fasciculus lateralis.* 3) *Fasciculus medialis.* 4) *Fasciculus dorsomedialis mihi.* 5) *Tr. preoptico-opticus.*

For the determination of the central connections of these components my material, consisting only of Weigert series, is not fully adequate, since it

[1]) Moreover, the tract was demonstrated by me in WEIGERT series of several species of teleosts, thus in Trachinus draco, Osmerus eperlanus, Exocoetus spec., Scardinius erythrophthalmus, Leuciscus rutilus, Scomber scombrus, and Ophiocephalus spec.

does not bring out the finer connections established by collaterals. However, the most important facts concerning the subject are revealed in my series. Thus it is easy to confirm that the fasciculus dorsalis and lateralis mainly terminate in the tectum opticum, as maintained by BELLONCI and others. The pronounced atrophy of the geniculate body on the side where the optic tract is lacking, indicates that the two fascicles under discussion likewise must be intimately connected with this nucleus, a connection which has been demonstrated before, thus by HOLMGREN in Golgi preparations. HOLMGREN, furthermore, observed that the fibres of the fasciculus dorsalis before entering the tectum discharge numerous collaterals which terminate amongst dendrites from the cells of the dorsal thalamus (eminentia thalami of HOLMGREN), a connection which he called tr. optico-eminentialis. — As to the ending of the fasciculus medialis caudally the fibres of this bundle cannot be differentiated from those of the fasciculus lateralis. Like the latter component the fasciculus medialis presumably terminates in the tectum. The fasciculus dorsomedialis courses caudad through the dorsal thalamus, and disappears in the dorsal part of the tegmentum mesencephali, near the nucleus of the posterior commissure. Whether the fascicle discharges collaterals in the dorsal thalamus could not be determined. As regards the tr. preoptico-opticus we know from HOLMGREN's observations that it constitutes, at least partly, an efferent, retinopetal component of the optic nerve.

It is seen then that the optic tract in teleosts beside its main field of termination in the tectum has important connections with the dorsal thalamic nuclei, not only the geniculate body, but the dorsomedial division of this region as well. A portion of the optic fibres also end dorsally in the tegmentum mesencephali.

To attempt a comparison with the conditions found in higher vertebrates is perhaps premature, since the homologies of the different diencephalic and mesencephalic nuclei in teleosts are still but imperfectly understood. The limited space, furthermore, does not permit of a detailed discussion of these questions. However, I should like in all brevity to call attention to the close correspondence of the conditions in teleosts and amphibians. HERRICK (1925) in his very interesting and instructive paper on the optic tract and centres of Amblystoma and the frog distinguishes: (a) the *marginal optic tract,* (b) the *axial optic tract,* and (c) the *tr. opticus accessorius posterior.*

The marginal optic tract after decussation spreads over almost the entire lateral surface of the thalamus, dorsally incompletely separating into medial and lateral fascicles, both of which connect with the tectum mesencephali. Golgi sections reveal that the marginal optic fibres are in functional relation with extensive areas of the dorsal thalamus, discharging collaterals as they sweep across the lateral surface. Terminals and collaterals are especially numerous in three regions, viz., (1) nucleus preopticus, (2) corpus geniculatum laterale, and (3) nucleus anterior superior of BELLONCI. The marginal optic tract clearly corresponds to the fasciculus dorsalis and

lateralis of teleosts. The latter fascicles both terminate in the tectum opticum, and we have seen above that they are connected also, like the marginal tract of amphibians, with extensive areas of the dorsal thalamus. One of these connections is with the lateral geniculate body, and obviously homologous to the optico-geniculate connection in amphibians. The second important optic connection within the dorsal thalamus was demonstrated by HOLMGREN (his tr. optico-eminentialis). There is a striking similarity between this optic centre in the dorsal thalamus of teleosts and the nucleus anterior superior of BELLONCI in amphibians. Thus (a) both nuclei occupy a position in the rostral part of the dorsal thalamus, immediately ventral to the habenula. (b) They have extensive optic connections. (c) The cells receive strong olfactory impulses (through the tr. olfacto-habenularis lateralis and other tracts). Consequently both nuclei constitute important olfacto-optic correlation centres.

The axial optic tract of Amblystoma decussates dorsally in the chiasma, in this respect resembling the fasciculus medialis of teleosts. HERRICK traced the tract under discussion in the deeper levels of the white substance as far as the lateral forebrain bundle. Beyond this point the course of the fibres is obscure, but in some preparations the tract could be traced "with tolerable precision" into the tectum. Thus the course of the axial tract closely resembles that of the fasciculus medialis tr. optici of teleosts.

The tr. opticus accessorius posterior (tr. opticus basalis) of Amblystoma from the posterior border of the chiasma sweeps along the ventrolateral surface of the brain to end in a superficial neuropil (nucleus ectomamillaris) lateral to the tuberculum posterius. The homologue of the basal optic tract I could not demonstrate in teleosts. HOLMGREN says (op. cit., p. 224) : „Die beiden Tracti preoptico-optici posteriores vertreten also hintere basale Wurzel des Opticus". However, these tracts cannot be regarded as homologous with the tr. opticus basalis of higher vertebrates. They resemble, it seems to me, much more closely the optic connections with the preoptic nucleus in Amblystoma, which HERRICK compares with the tr. opticus accessorius anterior of BOCHENEK.

A tract comparable with my fasciculus dorsomedialis tr. optici hitherto has not been demonstrated in amphibians.

Upon the whole it is seen that there exists a great similarity between teleosts and amphibians in regard to the distribution and the connections of the fibres of the optic tracts.

Summary.

An investigation of the primary optic system of teleostean fishes, primarily based on the examination of a specimen of Idus idus, the left eye of which was gone by disease, causing a complete degeneration of the right optic tract, has led to the conclusion that the optic tract of teleosts consists of five components, viz., (1) *Fasciculus dorsalis.* (2) *Fasciculus*

lateralis. (3) *Fasciculus medialis.* (4) *Fasciculus dorsomedialis.* (5) *Tr. preoptico-opticus.*

As to the ending of the optic fibres it is evident that the tectum opticum is the main field of termination. However, there exist also extensive optic connections with the dorsal thalamus, not only with the geniculate body, but with the rostral part of the dorsal thalamus as well. The latter optic nucleus, which forms an important olfacto-optic correlation centre (as maintained by HOLMGREN), in many respects resembles the nucleus anterior superior of BELLONCI in amphibians. A smaller bundle of optic fibres (fasciculus dorsomedialis mihi, probably already seen by WALLENBERG,) terminates dorsolaterally in the tegmentum mesencephali. The fibres of the tr. preoptico-opticus, partly at least, are efferent in nature, originating in the preoptic nucleus. The tract may perhaps be compared with the tr. opticus accessorius anterior, described by BOCHENEK (1908) in higher vertebrates. A homologue of the tr. opticus basalis was not found.

The tr. isthmo-opticus (FRANZ) and the fibrae tectales n. optici (KRAUSE), formerly held to represent efferent, tecto-retinal connections, do not constitute components of the primary optic system. The former is an isthmo-tectal tract, as maintained by KUDO, and therefore better named tr. isthmo-tectalis, as done here, following this investigator. The latter is a component of the postoptic commissure complex. Other efferent, retinopetal optic components than the tr. preoptico-opticus were not demonstrated. But it cannot be excluded, on the other hand, on the basis of my material, that there exists an olfacto-optic connection as maintained by HOLMGREN, and that efferent fibres may run mingled with the efferent fibres of the dorsal and lateral fascicles [1]).

A comparison between the primary optic system of teleosts and amphibians reveals striking similarities in the main features.

In concluding I wish to express my best thanks to Professor KAPPERS for placing all the facilities of his Institute at my disposal, and to Drs. J. L. ADDENS for his kind and very helpful criticism.

Anatomical Institute, University of Oslo.

September 1929.

BIBLIOGRAPHY.

BELLONCI, J., (1888), Ueber die centrale Endigung des Nervus opticus bei den Vertebraten. Zeitschr. wiss. Zool., Bd. 47, S. 1—46.

BOCHENEK, A., (1908), Ueber zentrale Endigungen des Nervus opticus. Bull. de l'Acad. des Sc. de Cracovie, p. 91—95.

CAJAL, S. RAMON Y, (1899), Die Structur des Chiasma opticum. Leipzig (Barth).

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[1]) The space does not allow to enter into a discussion of the experiments of AREY, who offers physiological proof for the existence of efferent fibres in the optic nerve of Amiurus, (L. B. AREY (1916)), The function of the efferent fibers of the optic nerve of fishes. Journ. Comp. Neur. Vol. 26, p. 213—245).

FRANZ, V., (1912), Beiträge zur Kenntnis des Mittelhirns und Zwischenhirns der Knochenfische. Fol. Neuro-Biol., Bd. **6**, S. 402—441.

HERRICK, C. JUDSON, (1925). The amphibian forebrain. III. The optic tracts and centers of Amblystoma and the frog. Journ. Comp. Neur., Vol. **39**, p. 433—489.

HOLMGREN, N. (1920), Zur Anatomie und Histologie des Vorder- und Zwischenhirns der Knochenfische. Acta Zool., Vol. **1**, p. 137—315.

KAPPERS, C. U. ARIËNS, (1921). Die vergleichende Anatomie des Nervensystems der Wirbeltiere und des Menschen. II. Haarlem (De Erven F. Bohn).

KUDO, K., (1923), Contributions to the knowledge of the brain of bony fishes. Proc. Roy. Academy Amsterdam, Vol. **26**, p. 65—78.

KRAUSE, K. (1898), Experimentelle Untersuchungen über die Sehbahnen des Goldkarpfens (Cyprinus auratus). Arch. mikr. Anat., Bd. **51**, S. 820—838.

LUBSEN, J., (Unpublished, see Kappers' Textbook, II, p. 1293).

WALLENBERG, A., (1913). Beitrag zur Kenntnis der Sehbahnen der Knochenfische. Le Névraxe, Vol. **14**, p. 251—275.

Chemistry. — *Die Pseudokomponenten des Wasserstoffs.* III. Von Prof. A. SMITS. (Communicated by Prof. P. ZEEMAN).

(Communicated at the meeting of October 26, 1929).

In meiner letzten Abhandlung über diesen Gegenstand [1]) zeigte ich, dass das innere Gleichgewicht zwischen den zwei Molekülarten α und β in den Grundzuständen α_0 und β_1 sich für verschiedenen Temperaturen, nach der Formel

$$ln\, K_{Gr} = -\frac{Q}{RT} + C$$

oder nach der BOLTZMANN'schen Gleichung

$$\frac{N_{\beta_1}}{N_{\alpha_0}} = \frac{p_{\beta_1}}{p_{\alpha_0}} \varepsilon^{-\frac{Q}{RT}}$$

berechnen lässt, wenn man für Q, den durch BEUTLER [2]) aus den spektroskopischen Messungen von HORI [3]) berechneten Wert 329 Cal., und für C den aus den statistischen Gewichten folgenden Wert 2,196 einsetzt.

Man sieht dabei sofort, dass der Maximumwert

$$(K_{Gr})_{T_\infty} = 9 \quad \text{ist.}$$

Es wurde weiter gezeigt dass das totale innere Gleichgewicht sich nicht so leicht angeben lässt, weil man dann nach DENNISON [4]) den verschiedenen Rotationszuständen, deren Verhältnis sich mit der Temperatur ändert, Rechnung tragen muss.

Aus dem durch HORI auf spektroskopischem Wege gefundenen Wert des Trägheitsmoments für das Wasserstoffmolekül lässt sich die Energie der verschiedenen Rotationszustände desselben Grundvibrationszustandes berechnen.

Die Energie des Wasserstoffmoleküls in dem Rotationszustand m und in dem Grundvibrationszustand A_0 ist dann

$$E^m_{A_0} = Bm^2 + \beta m^4 \quad \text{worin} \quad B = \frac{h^2}{8\pi^2 I} = 57{,}770 \text{ cm}^{-1}$$

$$\beta = 0{,}0165 \quad \text{während} \quad m = {}^1/_2,\ 1^1/_2,\ 2^1/_2 \text{ u.s.w.}$$

(Nach der Theorie von SCHRÖDINGER ist

$$E^m_{A_0} = B'.\, m\,(m+1)$$

in welcher Formel $m = 0,\ 1,\ 2,\ 3 \ldots.$)

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[1]) These Proceedings **32**, 951, (1929).
[2]) Z. f. Phys. **50**, 585 (1928).
[3]) Z. f. Phys. **44**, 850 (1927).
[4]) Proc. Roy. Soc. A, **115**, 483 (1927).

Aus den experimentellen Resultaten HORI's hat BEUTLER nun berechnet, dass die Energiedifferenz zwischen den Rotationszuständen 0 und 1 d.h. zwischen den Grundzuständen der beiden Molekülarten, also zwischen α_0 und β_1, **329** Cal. beträgt, was wir auf folgende Weise schreiben können:

$$E_{A_0}^{\beta_1} - E_{A_0}^{\alpha_0} = 329 \text{ Cal.}$$

Für den Uebergang zwischen den α-Rotationszuständen z.B. von α_0 in α_2 resp. α_0 in α_4 gilt:

$$E_{A_0}^{\alpha_2} - E_{A_0}^{\alpha_0} = 985 \text{ Cal.}$$

und

$$E_{A_0}^{\alpha_4} - E_{A_0}^{\alpha_0} = 3300 \text{ Cal.}$$

Für den Uebergang zwischen den β-Rotationszuständen z.B. von β_1 in β_3 resp. β_1 und β_5 bekommt man

$$E_{A_0}^{\beta_3} - E_{A_0}^{\beta_1} = 1646 \text{ Cal.}$$

und

$$E_{A_0}^{\beta_5} - E_{A_0}^{\beta_1} = 4630 \text{ Cal.}$$

Nun ist es möglich aus diesen Energie-Werten bezüglich des Grundzustandes bei Einführung der statistischen Gewichten der verschiedenen Zustände die Total-zusammensetzung, in total α und total β, also den Prozentgehalt der Pseudokomponenten α und β zu bestimmen.

Für einen bestimmten Rotationszustand m gilt nämlich:

$$\frac{N_m}{N} = \frac{p_m \cdot \varepsilon^{-\frac{E_m}{RT}}}{\Sigma\, p_m \cdot \varepsilon^{-\frac{E_m}{RT}}}$$

wo N_m = Anzahl Moleküle in dem Rotationszustand m,

E_m = die Energie dieser Zustände,

p_m = das statistische Gewicht des Zustandes m, und N = total Anzahl Moleküle.

Aus dieser BOLTZMANN'schen Verteilung und auch aus der experimentell gefundenen Verlauf der spezifischen Wärme von z. B. α (Para)-Wasserstoff folgt, dass auch bei niedrigen Temperaturen höhere Rotationszustände von den Pseudokomponenten vorkommen. Bei Einführung der statistischen Gewichte 1 : 3 für die α und β-Zustände findet man, dass, während das totale innere Gleichgewicht bei $T = 0$ ganz an der α-Seite liegt, dieses bei höheren Temperaturen sich nach der β-Seite verschiebt und asymptotisch den Limitwert 25 % α und 75 % β nähert, welcher Wert theoretisch erst bei $T = \infty$ erreicht wird. Also,

$$(K_{tot})_{T_\infty} = 3.$$

Es ist jetzt interessant zu sehen, wie die Gleichgewichtskonstante des

inneren Gleichgewichts zwischen den *Grundzuständen* α_0 und β_1 also $K_{Gr} = \frac{(\beta_1)}{(\alpha_0)}$, und die Gleichgewichtskonstante des totalen inneren Gleichgewichts $K_{tot} = \frac{(\alpha)}{(\beta)}$, sich mit der Temperatur änderen.

Man findet folgendes.

Inneres Gleichgewicht zwischen den Grundzustände.

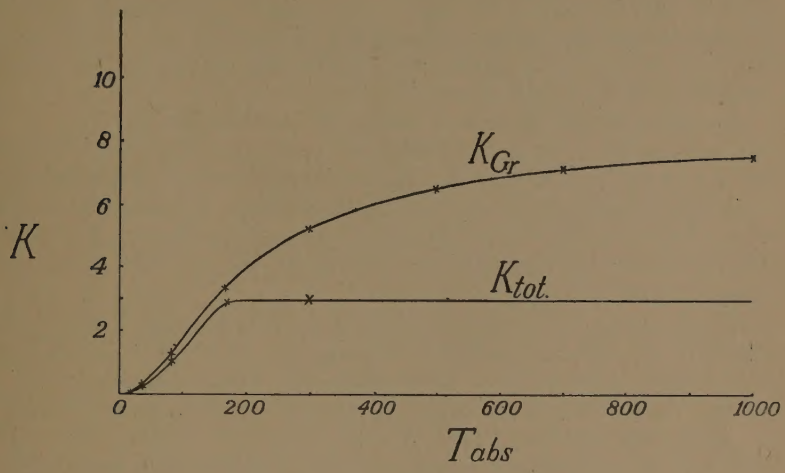
T_{abs}	$K_{Gr} = \frac{(\beta_1)}{(\alpha_0)}$
21.2°	0.004
42.5°	0.18
85°	1.28
170°	3.40
300°	5.18
500°	6.48
700°	7.10
1000°	7.63
2000°	8.29
∞	9

In der unterstehenden Tabelle ist neben den Prozentgehalt bezüglich aller α (Para) und aller β (Ortho)-Zustände die Konstante des totalen inneren Gleichgewichts angegeben.

Totales Inneres Gleichgewicht

T_{abs}	Totalzusammensetzung		$K_{tot} = \frac{(\beta)}{(\alpha)}$
	α (Para)	β (Ortho)	
21.2°	99.7	0.3	0.003
42.5°	85.8	14.2	0.17
85°	48.0	52.0	1.08
170°	25.3	74.7	2.95
300°	25.1	74.9	2.99
1000°	25+	75−	3−
2000°	25+	75−	3−
∞	25	75	3

Wenn wir diese Resultate grafisch darstellen, so bekommen wir unterstehende Figur.



Aus dieser Figur sehen wir nochmals deutlich, dass, während die Konstante des totalen inneren Gleichgewichts oberhalb $300°_{abs}$ sich beinahe nicht mehr ändert, die Konstante des inneren Gleichgewichts zwischen den Grundzuständen immer noch steigt.

Die immer stärkere Näherung dieser Kurven speziell unterhalb $170°_{abs}$ sagt, dass der Wasserstoff unterhalb dieser Temperatur sich immer stärker überwiegend in den Grundzuständen befindet, während bei höheren Temperaturen die höheren Rotationszustände immer mehr in den Vordergrund treten.

Diese Betrachtungen zeigen wie komplex und wie eigentümlich die Komplexität des Wasserstoffs ist.

Es ist uns, wie in der Übersicht in der Z. f. phys. Chemie **129**, 33 (1927) gezeigt wurde, schon gelungen bei einigen sogenannten einfachen Stoffen die Komplexität zu beweisen. Dieses wurde erreicht durch darzutun, dass die flüssige oder feste Phase eines einfachen Stoffes sich, unter bestimmten Umständen, wie die Phase eines mehr komponentigen Systems verhalten kann, und in dem Fall von SO_3 war es selbst möglich einen Teil des Pseudo-systems zu bestimmen und darzutun, dass in diesem Pseudo-system Verbindungen auftreten. [1])

Oft liegt es auf der Hand anzunehmen, dass die eine Pseudo-komponente ein Polymeres der anderen ist. In anderen Fällen scheint eine Isomerie am Wahrscheinlichtsten zu sein, und in 1923 wies Verfasser

[1]) Seit einige Jahre studiert Verfasser die Halogenen, und in der letzten Zeit auch den Stickstoff und Ammoniak in der selben Richtung. Er hofft bald seine Resultate sukzessive mitteilen zu können.

auf die Möglichkeit von der Existenz verschiedener Molekülarten mit viel feineren Unterschieden als bei gewöhnlicher Isomerie angenommen wird.

Mann konnte aber in den untersuchten Fällen noch nicht mit Sicherheit sagen, zu welcher Kategorie der Unterschied zwischen den verschiedenen Molekülarten gehörte.

Das Eigentümliche ist nun dieses, dass der einfachste Stoff, den wir kennen, der Wasserstoff, sich nicht nur vollkommen nach der Theorie der Allotropie verhält, sondern, dass hier der Unterschied zwischen den verschiedenen Molekülarten, dank sei der Arbeit von HEISENBERG [1]), HUND [2]) und DENNISON [3]) mit Sicherheit angegeben werden kann. Dieser Unterschied ist ein feiner Unterschied, welchen wir eine feine Isomerie nennen können.

Es ist erwünscht hier noch darauf hinzuweisen, dass der Wasserstoff eigentlich ein sehr verwickeltes System ist, weil die verschiedenen Rotationszustände, jeder der zwei Molekülarten, wie Pseudokomponenten aufgefasst werden können, sodass der Wasserstoff eigentlich einem Pseudomultär-System angehört. Weil aber die Übergänge zwischen den verschiedenen Rotationszuständen von dem selben Grundzustande warscheinlich immer sehr rasch stattfinden, lässt sich der Wasserstoff pseudobinär auffassen.

Wenn man die bei dem Wasserstoff erhaltenen Resultate übersieht, so erhebt sich die Frage in welchem Zustande der gewöhnliche Wasserstoff sich befindet. Oberhalbs 170°_{abs} verschiebt sich das totale innere Gleichgewicht nur sehr wenig und liegt dann ungefähr bei 25 % α (Para) und 75 % β (Ortho). Hieraus folgt, dass wenn man z.B., bei gewöhnlicher Temperatur, Wasserstoff von ungefähr dieser Zusammensetzung in Händen hat, man noch nicht sagen kann, dass dieser Wasserstoff sich in innerem Gleichgewicht befindet. Dieser Wasserstoff wurde ein Nichtgleichgewichtszustand sein können, welcher z.B. bei einer anderen Temperatur, oberhalb 170°_{abs}, einen inneren Gleichgewichtszustand darstellte.

Mittelst sehr genauer Messungen wird man dieser Frage hoffentlich näher treten können, und was auch sehr interessant ist, man wird Wasserstoff verschiedener Herkunft untersuchen können um zu sehen, ob, oder in wie weit, diese Produkte eine verschiedene Zusammensetzung besitzen.

Es leuchtet ein, dass, auch wenn man Wasserstoff hat, der sich nach längerer Zeit bei gewöhnlicher Temperatur in innerem Gleichgewicht gesetzt hat, dieser Zustand, der langsamen Einstellung dieses Gleichgewichts wegen, bei Abkühlung resp. Verflüssigung in einen Nicht-Gleichgewichtszustand d.h. in einen Gemisch von praktisch der selben

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[1]) Z. f. Phys. **41**, 239 (1927).

[2]) Z. f. Phys. **42**, 93 (1927).

[3]) Proc. Royal Soc. A. **115**, 483 (1927).

Zusammensetzung übergehen wird. Nur bei Anwendung geeigneter Katalysatoren oder Bestrahlung wird es vielleicht möglich sein zu erreichen, dass der Wasserstoff, während einer langsamen Temperaturänderung, so wie während des Siedens, der Verdampfung, der Schmelzung, resp. Verflüssigung, und Erstarrung in innerem Gleichgewicht bleibt.

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Amsterdam, 17 Oct. 1929.